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Numerical study of a model of steady magnetohydrodynamics free convection

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Abstract: Several engineering phenomena require the understanding of MHD flows and thermal effects related to complex flows. For instance, example is metallurgy, crystal growth and aerospace engineering problems. The present work is an effort towards numerical investigation of such flows. We examine the steady three-dimensional flow over a semi infinite permeable surface. The surface is embedded in a porous medium. The mathematical model derived from the incompressible Navier-Stokes equations and Maxwell's equation comprises of nonlinear ODEs subject to boundary conditions. The nonlinearity poses a major challenge for the solution of model. Numerical simulations are carried out for the proposed MHD flow for different values of important parameters. The study presents analysis of the quantities like velocities, temperature and stream function. Hence the flow behavior is investigated in context of hard physical conditions. Results are shown graphically to ensure the accuracy and effectiveness of applied algorithm.

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1. Introduction

Magnetohydrodynamics (MHD) deals with the flow of electrically conducting fluids and plasmas in the presence of magnetic field. The study of MHD flows has been one of an attracting area for researchers and engineers over decades. Its worth lies in the vast number of applications from engineering, aerodynamics, for instance, power generators, MHD accelerators, transmission lines and electric transformers. These flows are mathematically described by the coupled Navier-Stokes equations coupled with the Maxwell's equations. The flow also encounters and induced magnetic field and electric currents.

There are numerous benchmark problems in the literature that reflect the importance of MHD flows. For instance, one can find the research carried out in the literature (see [1 – 14]). Numerical solutions play a vital role in understanding these flows. The work carried ny Botella and Peyret [2] is pioneered in the area of incompressible viscous flows. Eruk. [6] presented 2D flow analysis and studied iso-thermal phenomena.

Boundary layer flows have been studied first time in [13] and [14] by Skiadis. The study was extended by Erickson et al. [7] who considered the injection/suction effects also. Numerical investigations have been carried out by Hamad et al. [8] and Chen [5] for the MHD convection boundary layer problems. Prasad et al. [12] presented a numerical analysis for heat and mass transfer problem of flow over a vertical plate in non-Dacry porous medium.

In order to solve the mathematical models of such flows, approximate analytical techniques have been applied successfully. These include Homotopy perturbation method, Homotopy analysis method, and differential transform method. These methods give series solutions which are truncated after few terms. Thus getting a more accurate solution is one of important task. The numerical techniques include finite difference methods, shooting method, finite element methods and finite volume methods. These methods are applied according to the physics, geometry and the equations involved.

The proposed research problem is represented by a set of non-linear partial differential equations (PDEs) with appropriate boundary conditions. The non-linearity of the mathematical model poses great challenge for solving the model. The set of PDEs is reduced to a set of coupled nonlinear ODEs. The boundary layer scale analysis, Lie group analysis and similarity transformations are used for reduction of the model to ODEs. Accordingly the boundary conditions are modified. Hence a set of coupled nonlinear ODEs is obtained subject to the boundary conditions.

Finally the set of ODEs is solved by shooting method and Homotopy Analysis method. The Newton iteration technique is incorporated to modify the shooting method for the present problem. It is worth mentioning that this modified shooting method is presented first time in the literature for the solution nonlinear coupled higher orders ODEs.

2. Mathematical formulation:

Consider a steady, laminar three dimensional free convective boundary layer flow of an electrically conducting fluid over a semi-infinite inclined surface. The surface is embedded in a porous medium and experiences linear stretching in x-direction with velocity *bx*. The y-axis is at angle $\propto$ with the horizontal line while z-axis is normal to plane surface. A uniform magnetic field is applied in y- direction that accounts for the magnetic effects in xz plane. All dependent variables will be independent of the y-direction; including the viscous term, Darcy number and magnetic force. The flow is summarized in a set of following mathematical relations:

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0, \qquad (2.1)$$

$$u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}-v\frac{\partial^2 u}{\partial z^2}-g\beta(T-T_\infty)\cos\propto+\frac{\sigma B_0^2}{\rho}u+\frac{v}{k}u=0, \qquad (2.2)$$

$$w\frac{\partial v}{\partial z}-v\frac{\partial^2 v}{\partial z^2}-g\beta(T-T_\infty)\sin\propto+\frac{v}{k}v=0, \qquad (2.3)$$

$$w\frac{\partial w}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial z}-v\frac{\partial^2 w}{\partial z^2}+\frac{\sigma B_0^2}{\rho}w+\frac{v}{k}w=0, \qquad (2.4)$$

$$w\frac{\partial T}{\partial z}-\frac{v}{Pr}\frac{\partial^2 T}{\partial z^2}-\frac{Q_o}{\rho C_p}(T-T_\infty)=0. \qquad (2.5)$$

Where u, v, w, p and T are the components of fluid velocity along the x, y and z axis, pressure and temperature respectively. The constant values include ρ, v, C_p k, g, b, T_∞, $\propto$ and P_r i.e., fluid density, kinematic viscosity, specific heat, thermal conductivity, gravitational acceleration, coefficient of thermal expansion, ambient temperature, the inclination angle and Prandtl number respectively. Moreover σ, B_o, Q_o denote the fluid electrical conductivity, magnetic induction and dimensional heat coefficient respectively.

The boundary conditions are

$$u(x,0)=bx,\quad v(x,0)=0,\quad w(x,0)=w_o,\quad \theta(x,0)=\theta_w(x)$$
$$u(x,\infty)=0,\quad v(x,\infty)=0,\quad w_z(x,0)=0,\quad \theta(x,\infty)=0, \qquad 2.6$$

taking $\theta=T-T_\infty$ and $\theta_w=T_w-T_\infty$ along the boundary surface z = 0.

The boundary value problem, described by equations (2.1) - (2.5) and the data (2.6), admits the following multi parameter group of symmetries:

$$x^*=x+\varepsilon(2c_1x)+O(\varepsilon^2),$$
$$z^*=z+\varepsilon\left(c_2x+\left(\frac{c_1}{2}\right)z\right)+O(\varepsilon^2),$$
$$u^*=u+\varepsilon(c_1u+h_3)+O(\varepsilon^2),$$
$$v^*=v+\varepsilon(c_1v+h_4)+O(\varepsilon^2),$$
$$w^*=w+\varepsilon\left(c_2u-\left(\frac{c_1}{2}\right)w+h_5\right)+O(\varepsilon^2),$$
$$p^*=p+\varepsilon(c_1p+h_6)+O(\varepsilon^2),$$
$$\theta^*=\theta+\varepsilon(c_1\theta+h_7)+O(\varepsilon^2).$$

2.7

We restrict to a case with no permeability of porous medium effect. Hence the transformation equations are of the following form:

$$x^* = e^{2\varepsilon}x, \quad z^* = e^{\frac{\varepsilon}{2}}z, \quad u^* = e^{\varepsilon}u, \; v^* = e^{\varepsilon}v, \; w^* = e^{-\frac{\varepsilon}{2}}w$$

$$p^* = e^{\varepsilon}p, \quad \theta = e^{\varepsilon}\theta.$$

Under this choice of the parameters, the self-similar solutions takes the form:

$$u(x,z) = k_1 x F'(\eta) + k_1 \; M(\eta),$$
$$v(x,z) = k_2 N(\eta),$$
$$w(x,z) = -k_3 F(\eta),$$
$$p(x,z) = k_4 G(\eta),$$
$$\theta(x,z) = k_5 H(\eta),$$
$$\eta = k_6 z,$$

Substitution of equations (2.8) into equations (2.1)-(2.6) results the non-similar transient equations given below:

$$F''' + \frac{\eta}{2}F'' + b\left(F F'' - F'^2 - \left(Ha^2 + \frac{1}{Da}\right)F'\right) = 0 \tag{2.9}$$

$$M'' + \frac{\eta}{2}M' + b\left(FM' - MF' - \left(Ha^2 + \frac{1}{Da}\right)M + H\right) = 0 \tag{2.10}$$

$$N'' + \frac{\eta}{2}N' + b\left(FN' - \frac{1}{Da}N + H\right) = 0 \tag{2.11}$$

$$G' + F'' + \frac{\eta}{2}F' + b\left(FF' - \left(Ha^2 + \frac{1}{Da}\right)F\right) = 0 \tag{2.12}$$

$$H'' + Pr\frac{\eta}{2}H' + Prb\,(FH' + QH) = 0 \tag{2.13}$$

The boundary conditions are transformed to following:

$$F(0) = F_w, \;\; F'(0) = 1, \;\; F'(\infty) = 0, \;\; M(0) = 0, \;\; M(\infty) = 0,$$
$$N(0) = 0, \;\; N(\infty) = 0, \;\; G(0) = 0, \;\; \theta(0) = 1, \;\; \theta(\infty) = 0 \tag{2.14}$$

2.1. Solving the problem:

The equations (2.9) - (2.13) are reduced to sets of first-order equations by defining new dependent variables $F^0, F^1, F^2, M^0, M^1, N^0, N^1, G^0, H^0$ and H^1. As a result, we obtained the following system of differential equations.

$$\frac{d^3F}{d\eta^3} + \frac{\eta}{2}\frac{d^2F}{d\eta^2} + b\left(F\frac{d^2F}{d\eta^2} - \left(\frac{dF}{d\eta}\right)^2 - \left(Ha^2 + \frac{1}{Da}\right)\frac{dF}{d\eta}\right) = 0$$

Let $F^0 = F$,

$$\frac{dF^0}{d\eta} = \frac{dF}{d\eta} = F^1,$$

$$\frac{d^2F^0}{d\eta^2} = \frac{d^2F}{d\eta^2} = F^2 \quad \text{and}$$

$$\frac{d^3F}{d\eta^3} = \frac{dF^2}{d\eta} = -\frac{\eta}{2}F^2 - b\left(F^0F^2 - (F^1)^2 - \left(Ha^2 + \frac{1}{Da}\right)F^1\right)$$

So, we have the system

$$\frac{dF^0}{d\eta} = F^1$$

$$\frac{dF^1}{d\eta} = F^2$$

$$\frac{dF^2}{d\eta} = -\frac{\eta}{2}F^2 - b\left(F^0F^2 - (F^1)^2 - \left(Ha^2 + \frac{1}{Da}\right)F^1\right) \tag{2.9}$$

$$\frac{d^2M}{d\eta^2}+\frac{\eta}{2}\frac{dM}{d\eta}+b\left(F\frac{dM}{d\eta}-M\frac{dF}{d\eta}-\left(Ha^2+\frac{1}{Da}\right)M+H\right)=0$$

Let $M^0=M$

$$\frac{dM^0}{d\eta}=\frac{dM}{d\eta}=M^1 \quad \text{and}$$

$$\frac{d^2M^0}{d\eta^2}=\frac{dM^1}{d\eta}=-\frac{\eta}{2}M^1-b\left(F^0M^1-M^0F^1-\left(Ha^2+\frac{1}{Da}\right)M^0+H\right)$$

So, we have the system

$$\frac{dM^0}{d\eta}=M^1$$

$$\frac{dM^1}{d\eta}=-\frac{\eta}{2}M^1-b\left(F^0M^1-M^0F^1-\left(Ha^2+\frac{1}{Da}\right)M^0+H\right) \qquad 2.10$$

$$\frac{d^2N}{d\eta^2}+\frac{\eta}{2}\frac{dN}{d\eta}+b\left(F^0\frac{dN}{d\eta}-\frac{1}{Da}N+H\right)=0$$

Let $N^0=N$,

$$\frac{dN^0}{d\eta}=\frac{dN}{d\eta}=N^1 \quad \text{and}$$

$$\frac{d^2N^0}{d\eta^2}=\frac{dN^1}{d\eta}=-\frac{\eta}{2}N^1-b\left(F^0N^1-\frac{1}{Da}N^0+H\right)$$

So, we have the system

$$\frac{dN^0}{d\eta}=N^1$$

$$\frac{dN^1}{d\eta}=-\frac{\eta}{2}N^1-b\left(F^0N^1-\frac{1}{Da}N^0+H\right) \qquad 2.11$$

$$\frac{dG}{d\eta}+\frac{d^2F}{d\eta^2}+\frac{\eta}{2}\frac{dF}{d\eta}+b\left(F\frac{dF}{d\eta}-\left(Ha^2+\frac{1}{Da}\right)F\right)=0$$

By supposing $G^0=G$, we have equation as under

$$\frac{dG^0}{d\eta}=-F^2-\frac{\eta}{2}F^1-b\left(FF^1-\left(Ha^2+\frac{1}{Da}\right)F^0\right) \qquad 2.12$$

$$\frac{d^2H}{d\eta^2}+Pr\frac{\eta}{2}\frac{dH}{d\eta}+Prb\left(F\frac{dH}{d\eta}+QH\right)=0$$

Let $H^0=H$,

$$\frac{dH^0}{d\eta}=\frac{dH}{d\eta}=H^1 \quad \text{and}$$

$$\frac{d^2H^0}{d\eta^2}=\frac{dH^1}{d\eta}=-Pr\frac{\eta}{2}H^1-Prb\left(F^0H^1+QH^0\right)$$

So, we have the system

$$\frac{dH^0}{d\eta}=H^1$$

$$\frac{dH^1}{d\eta}=-Pr\frac{\eta}{2}H^1-Prb\left(F^0H^1+QH^0\right) \qquad 2.13$$

Here F^0, M^0, N^0, G^0 *and* H^0 are equivalent to *F, M, N, G* and *H*. F^1, F^2, M^1, N^1 and H^1 are the first and second order derivatives of *F, M, N* and *H* with respect to η. The eight coupled first order ODEs can be readily solved in Matlab using the built in ODE45 solver. The ODEs are defined with in a Matlab m-file "freeconvect.m" where the function "Y = freeconvect (eta, X, Pr, Da, Ha) calculates the set of first derivatives.

$$Y = \begin{bmatrix} \frac{dF^0}{d\eta} & \frac{dF^1}{d\eta} & \frac{dF^2}{d\eta} & \frac{dM^0}{d\eta} & \frac{dM^1}{d\eta} & \frac{dN^0}{d\eta} & \frac{dN^1}{d\eta} & \frac{dH^0}{d\eta} & \frac{dH^1}{d\eta} \end{bmatrix}$$ with

$$X = \begin{bmatrix} F^0 & F^1 & F^2 & M^0 & M^1 & N^0 & N^1 & H^0 & H^1 \end{bmatrix}$$

A driver program is used to solve the ten coupled ODE's subject to a set of ten initial conditions "Xinit" over a finite range of η, "etaspan". In this case, the finite range chosen is $0 \leq \eta \leq 10$.

2.2. Solving coupled nonlinear equations:

Newton's method for solving nonlinear equations involves an iterative process of iteratively refining x, by a correction h,

$x_{i+1} = x_i + h$.

The increment h is calculated by linear extrapolation of the function, $f(x)$, to zero.

$0 = f(x_i) + \left(df/dx\right)_{x_i} h$.

This approach can be scaled up readily to solve the roots of coupled equations. For this system, the equations in matrix form are

$$\begin{bmatrix} F^2(\eta=0)_{i+1} \\ M^1(\eta=0)_{i+1} \\ N^1(\eta=0)_{i+1} \\ H^1(\eta=0)_{i+1} \end{bmatrix} = \begin{bmatrix} F^2(\eta=0)_i \\ M^1(\eta=0)_i \\ N^1(\eta=0)_i \\ H^1(\eta=0)_i \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix}.$$

Where $[h_1, h_2, h_3, h_4]^t$ is the solution to the equation

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} F^1(\eta=0)_i \\ M^0(\eta=0)_i \\ N^0(\eta=0)_i \\ H^0(\eta=0)_i \end{bmatrix} + \begin{bmatrix} \frac{dF^1(\eta=10)}{dF^2(\eta=0)}\big|_{H^1} & \frac{dF^1(\eta=10)}{dM^1(\eta=0)}\big|_{N^1} & \frac{dF^1(\eta=10)}{dN^1(\eta=0)}\big|_{M^1} & \frac{dF^1(\eta=10)}{dH^1(\eta=0)}\big|_{F^2} \\ \frac{dM^0(\eta=10)}{dF^2(\eta=0)}\big|_{H^1} & \frac{dM^0(\eta=10}{dM^1(\eta=0)}\big|_{N^1} & \frac{dM^0(\eta=10)}{dN^1(\eta=0)}\big|_{M^1} & \frac{dM^0(\eta=10)}{dH^1(\eta=0)}\big|_{F^2} \\ \frac{dN^0(\eta=10)}{dF^2(\eta=0)}\big|_{H^1} & \frac{dN^0(\eta=10}{dM^1(\eta=0}\big|_{N^1} & \frac{dN^0(\eta=10)}{dN^1(\eta=0)}\big|_{M^1} & \frac{dN^0(\eta=10)}{dH^1(\eta=0)}\big|_{F^2} \\ \frac{dH^0(\eta=10)}{dF^2(\eta=0)}\big|_{H^1} & \frac{dH^0(\eta=10)}{dM^1(\eta=0)}\big|_{N^1} & \frac{dH^0(\eta=10)}{dN^1(\eta=0)}\big|_{M^1} & \frac{dH^0(\eta=10)}{dH^1(\eta=0)}\big|_{F^2} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix}$$

This process is expressed using matrix and vector variables as

$X_{i+1} = X_i + h$ and $0 = F + \bar{K}H$

to achieve an iterative formula $X_{i+1} = X_i - \bar{K}^{-1}F$, where $\bar{K}$ is Jacobian matrix.

Although the sixteen derivatives, that comprise $\bar{K}$ cannot be expressed analytically, they can be estimated using two term forward finite divided differences. For example,

$$\frac{dF^1(\eta=10)}{dF^2(\eta=0)}\Big|_{H^1} = \frac{F^1(\eta=10)|_{F^2+\delta F,H^1} - F^1(\eta=10)|_{F^2,H^1}}{\delta F}.$$

$$\frac{dM^0(\eta=10)}{dF^2(\eta=0)}\Big|_{H^1} = \frac{M^0(\eta=10)|_{F^2+\delta F,H^1} - M^0(\eta=10)|_{F^2,H^1}}{\delta F} \ldots (2.25)$$

$$\frac{dN^0(\eta=10)}{dF^2(\eta=0)}\Big|_{H^1} = \frac{N^0(\eta=10)|_{F^2+\delta F,H^1} - N^0(\eta=10)|_{F^2,H^1}}{\delta F} \ldots (2.26)$$

$$\frac{dH^0(\eta = 10)}{dF^2(\eta = 0)}|_{H^1} = \frac{H^0(\eta = 10)|_{F^2+\delta F, H^1} - H^0(\eta = 10)|_{F^2, H^1}}{\delta F} \ldots (2.27)$$

To estimate the sixteen derivatives, the ODEs in Eq. (2.14) have to be solved for five pairs of initial values $\{F_i^{\,2}, M_i^{\,1}, N_i^{\,1}, H_i^{\,1}\}$, $\{F_i^{\,2} + \delta_F, M_i^{\,1}, N_i^{\,1}, H_i^{\,1}\}, \{F_i^{\,2}, M_i^{\,1} + \delta_M, N_i^{\,1}, H_i^{\,1}\}, \{F_i^{\,2}, M_i^{\,1}, N_i^{\,1} + \delta_N, H_i^{\,1}\}$ and $\{F_i^{\,2}, M_i^{\,1}, N_i^{\,1}, H_i^{\,1} + \delta_H\}$

where $\delta_F, \delta_M, \delta_N$ and δ_H are small numbers.

A Matlab code for solving this boundary value problem is developed. The ODEs are solved over the interval $0 \leq \eta \leq 10$ for five pairs of initial values, $\{F_i^{\,2}, M_i^{\,1}, N_i^{\,1}, H_i^{\,1}\}$, $\{F_i^{\,2} + \delta_F, M_i^{\,1}, N_i^{\,1}, H_i^{\,1}\}, \{F_i^{\,2}, M_i^{\,1} + \delta_M, N_i^{\,1}, H_i^{\,1}\}, \{F_i^{\,2}, M_i^{\,1}, N_i^{\,1} + \delta_N, H_i^{\,1}\}$ and $\{F_i^{\,2}, M_i^{\,1}, N_i^{\,1}, H_i^{\,1} + \delta_H\}$.

The resultant values of F^1, M^0, N^0 and H^0 at $\eta = 10$ for each pair of initial conditions is placed into an array (X1, X2, X3, X4 and X5) and sixteen derivatives that comprise $\bar{K}$ are calculated which is used to get the estimates of initial values using eq. 2.23.

2.3. Numerical Results

2.3.1. Problem 1. The presented model is simulated for following set of values.

$F(0) = 0,\ Ha = 0.0,\ Da = 100,\ Qa = 0.0,\ \Pr = 0.7.$

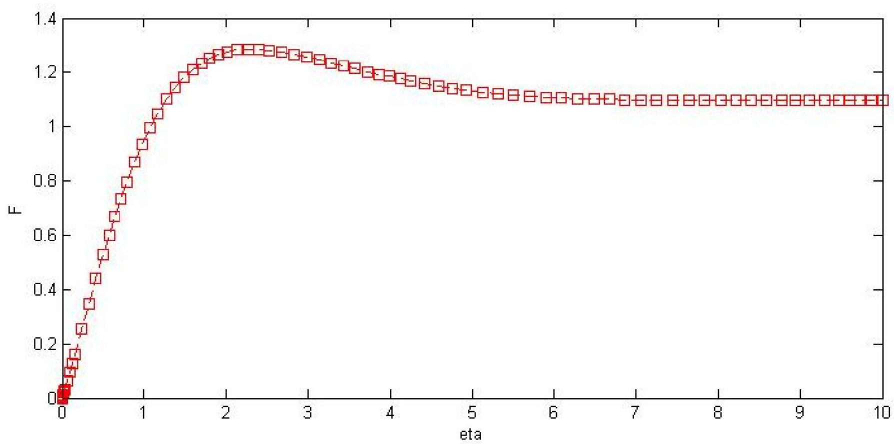


Figure 1.

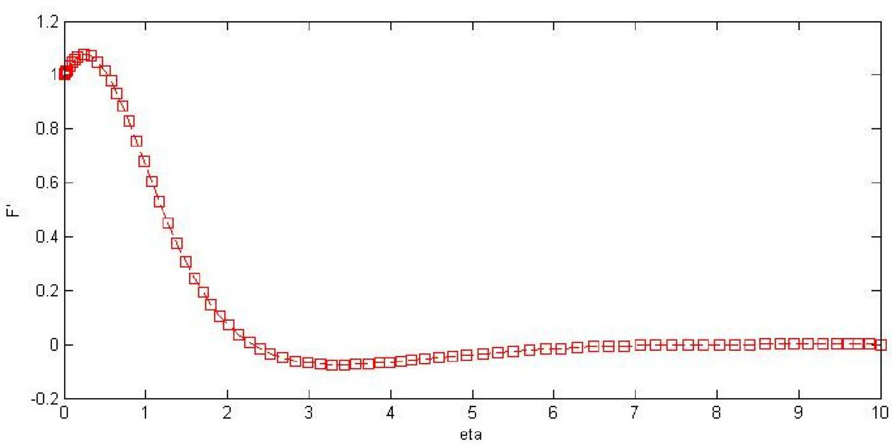
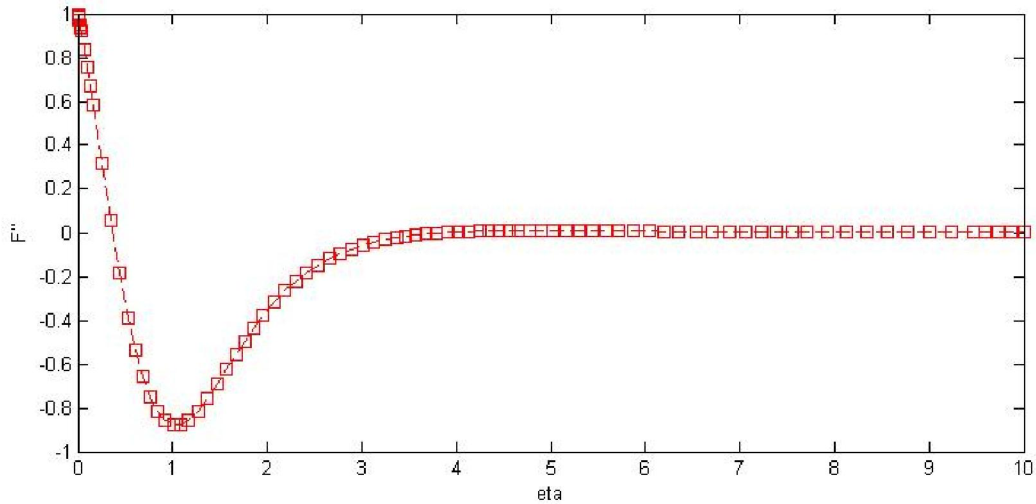


Figure 2.



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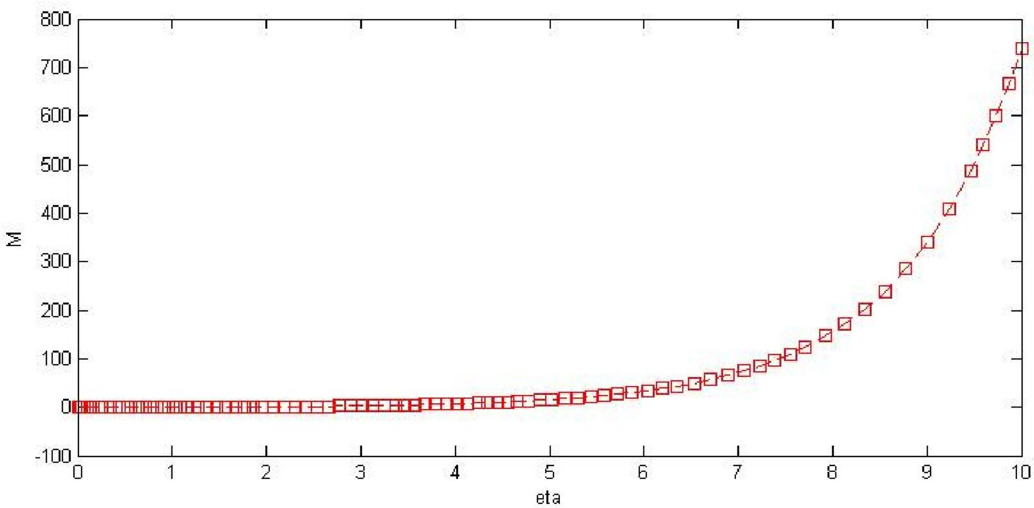

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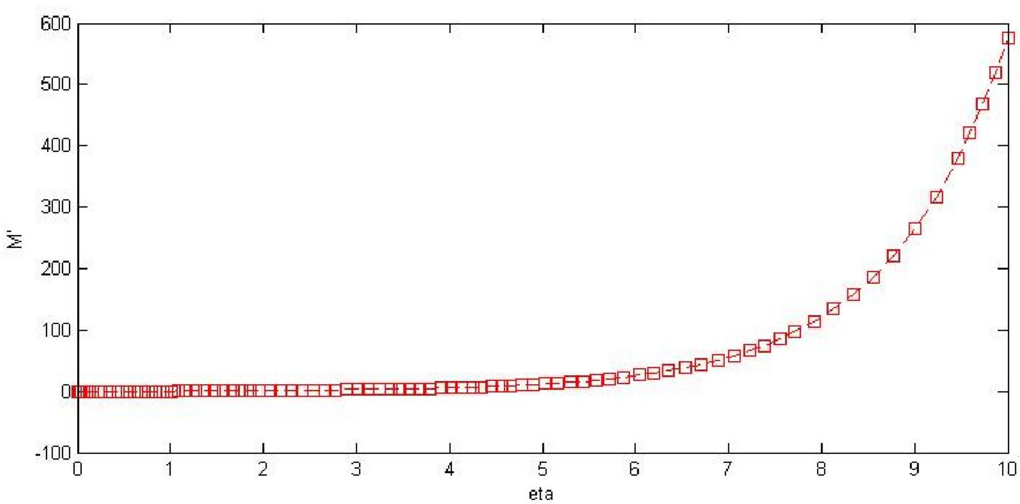

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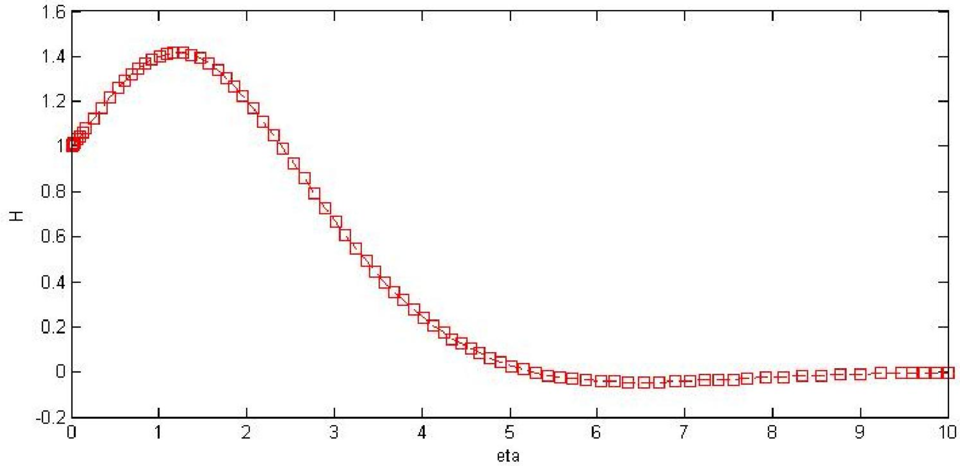

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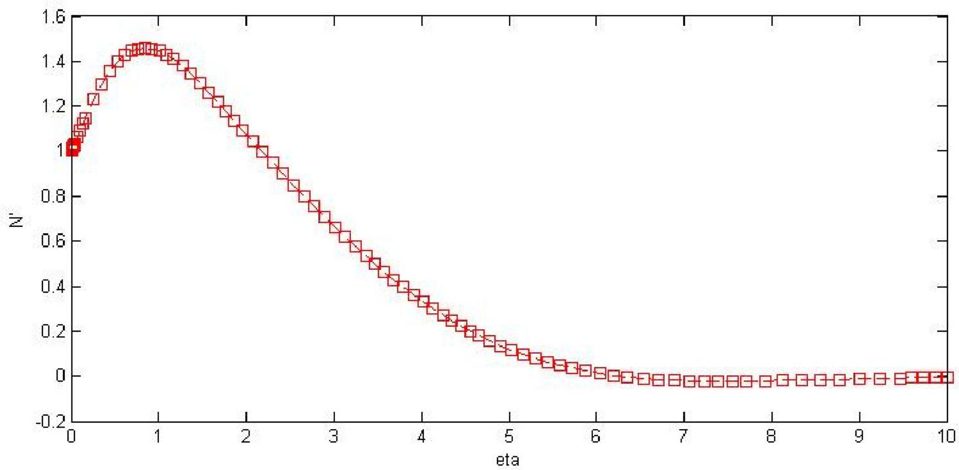


Figure 7.

2.3.2. Problem 2. The presented model is simulated for following set of values.

$F(0)=0,\ \ Ha=1.0,\ Da=100,\ Qa=0.0,\ \Pr=0.7.$

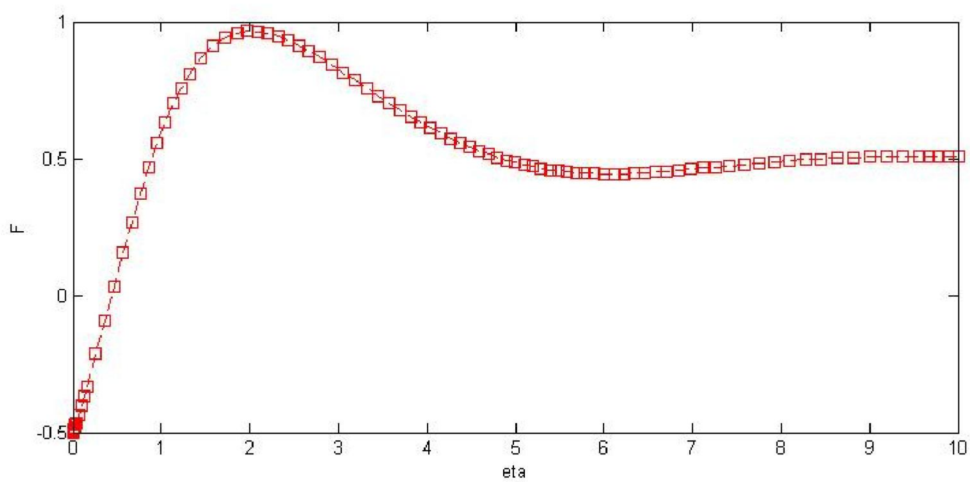


Figure 8.

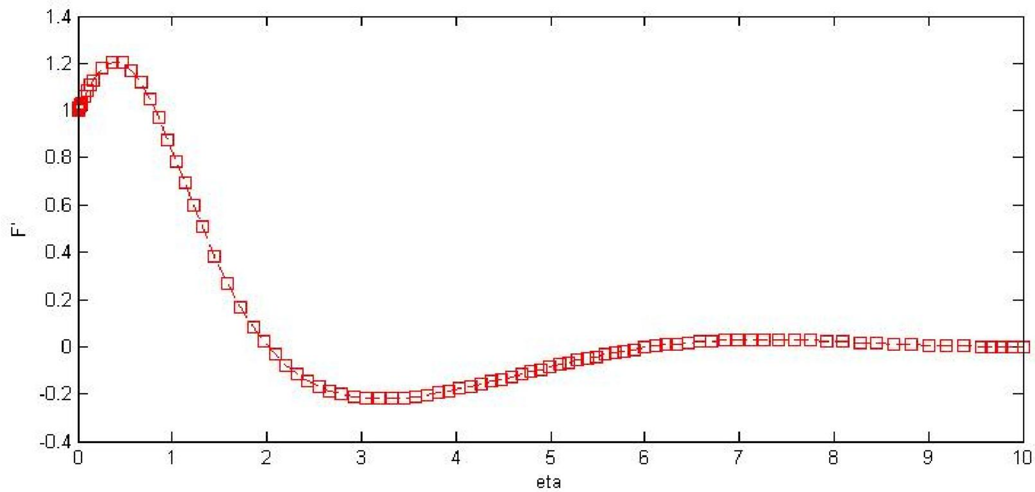
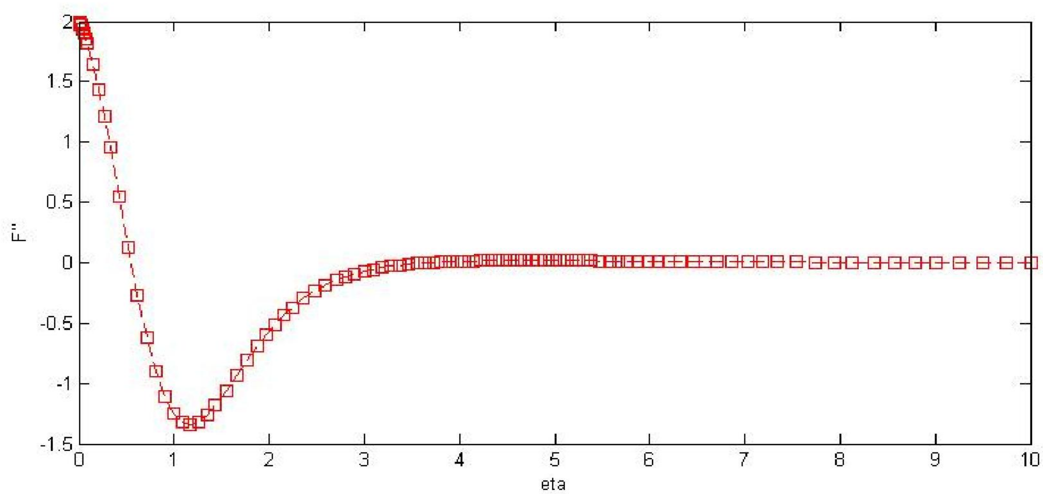


Figure 9.



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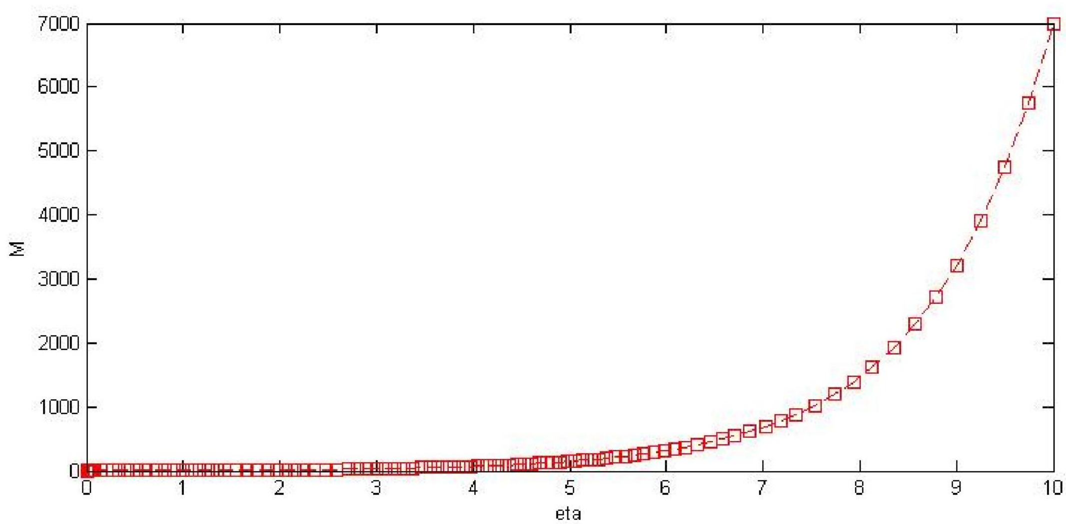

Figure 11.

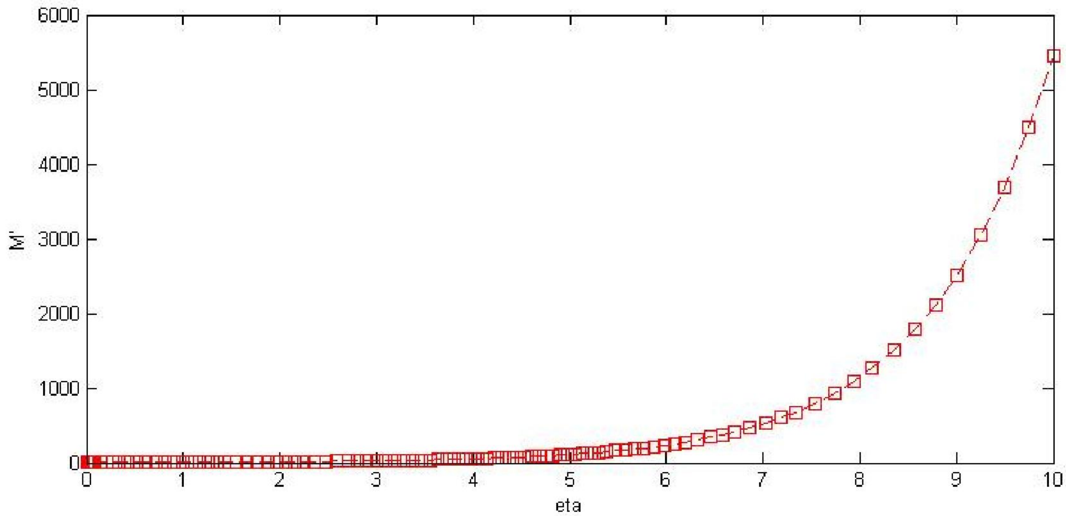

Figure 12.

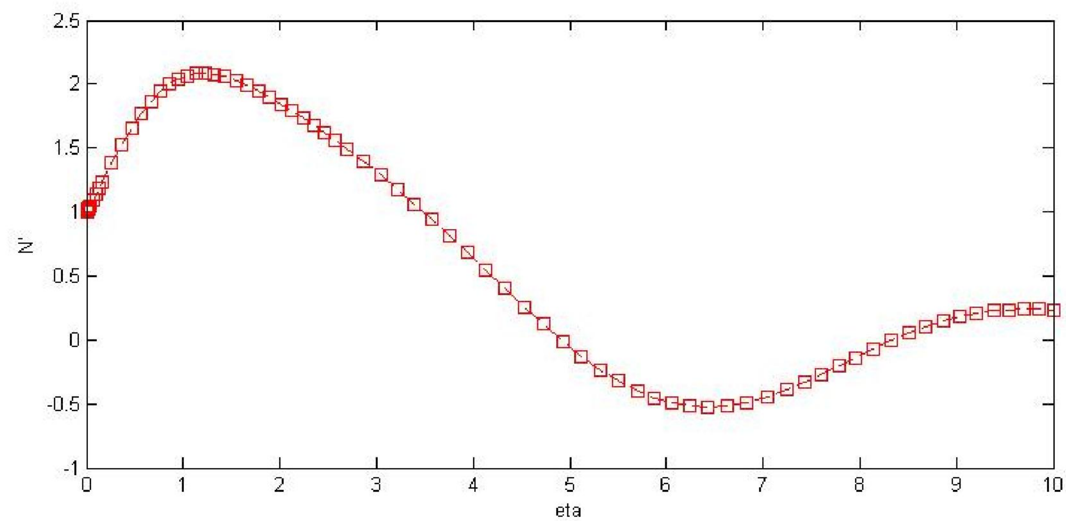

Figure 13.

3. Conclusions

The results are presented for the fluid's x-component of velocity $F^{'}$, y-component of velocity M, z-component of velocity N and for temperature H. The numerical results are obtained for two different values of Hartmann number Ha. While the values of rest of parameters which are Prandtl number Pr, Dacry number Da and heat generation/ absorption coefficient Q are fixed for both cases. Figure 1-13 clearly shows that change in the values of Hartmann number has a significant impact on fluid's behavior.

The aim was to develop a numerical algorithm for the presented model. It is seen that eighteen iterations were required to achieve the desired accuracy.

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