Modified formulation for tension stiffening effect on deflection of reinforced concrete rectangular beams

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Abstract: Deformations of reinforced concrete members are considered major design criteria. Design codes used to determine upper limits for deformations in the serviceability state, to ensure deformations are not noticeable Deflection of reinforced concrete beams is the most studied pattern of deformations in literature. Up till now evaluation of effective reinforced concrete beams inertia is still non-settled point. Design codes use various empirical forms for evaluating effective inertia. These different forms result in different deflection values. This research presents a modification for one of the most relevant formulae proposed for calculating the effective moment of inertia, which is an important parameter in calculating deflection of reinforced concrete beams. The research has adopted both experimental and analytical approaches to deal with the problem. An experimental program, composed of ten rectangular beams, tested in four points bend configuration has been accomplished. Beams of different stiffness, characteristic strength, yield stress, and reinforcement ratios have been tested. An analytical approach has also been presented through mathematical derivation of a formula for evaluating effects of tension stiffening on beam's moment of inertia. Results of the experimental program have been used to evaluate tension stiffening coefficient in the analytical approach. The generated formula has shown accepted correlation with experimental results. Moreover, influences of various structural parameters on tension stiffening have been studied via the experimental program.

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Key words: Reinforced concrete, beams, deflection, gross section inertia, cracked section inertia, effective moment of inertia, tension stiffening, reinforcement ratio, balanced reinforcement ratio.

1. Introduction

Deflection is a function of structural system, loading pattern, applied bending moment, and member's stiffness; which is by itself a function of member's length, modulus of elasticity and moment of inertia (the main argument of this paper). Evaluation of moment of inertia, of reinforced concrete members needs a special treatment; as it is dependent on the state of cracking of reinforced concrete element. Moment of inertia could be accurately evaluated for un-cracked (I_g) and cracked (I_{cr}) sections. The actual case of most reinforced concrete element is an intermediate situation between cracked and uncracked cases (according to crack pattern).

The un-cracked case could be treated as a composite section consisting of concrete and steel where each could contribute to the total moment of inertia by a load factor, which is its modulus of elasticity; while after cracking concrete has no contribution in resisting tensile stresses, so a portion of concrete is neglected and the other portion contributes by a factor which is its modulus of elasticity relative to that of reinforcement. The main argument in reinforced concrete is post-cracking because cracks take place in some sections only, so the beam is not cracked at all sections which means that some sections are cracked while others are not, that is called effect of tension stiffening effect. Figure (1) presents tension stiffening in the form of moment-curvature function for gross, cracked and effective inertia (I_{eff}) of reinforced concrete members in flexure.

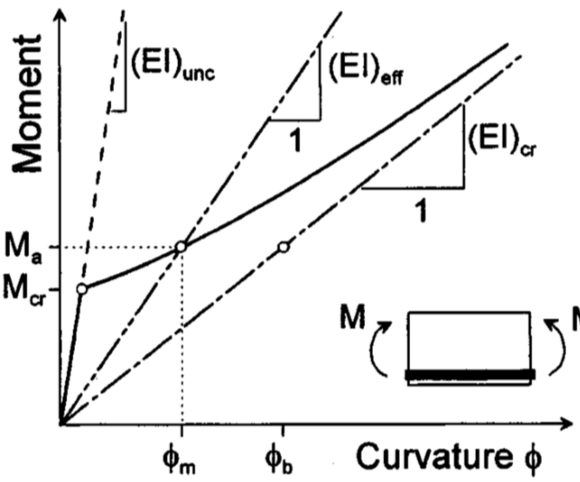


Figure (1) Tension stiffening in R.C. members

Many researchers proposed various formulation for evaluating effective moment of inertia. One of the most classical formulations for (I_{eff}), presented by İlkerKalkan[1], is that proposed by Branson, which consists of two portions one is a function and the gross and other is a function of the cracked inertia, as shown in equation (1).

$$I_{\text{eff}} = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left(1 - \left[\frac{M_{cr}}{M_a}\right]^3\right) I_{cr} \leq I_g \qquad (1)$$

Branson's equation is an empirical formula, where it has been calibrated with experimental data for rectangular beams with reinforcement ratio of 1.6%.

That is why it is efficiently used in case of medium to high reinforced beams.

Haider K et al[2]derived a comprehensive formula for deflection evaluation, based on experimental data, taking into consideration beam depth to span ratio, as shown in equation (2)& (3).

$$\Delta = \frac{CM_aL^2}{E_cI_{eff}}\zeta\Psi \quad (2)$$

Where

$$I_{\text{eff}} = \left(\frac{M_{cr}}{M_a}\right)^{\alpha} I_g\left(1-\frac{h}{L}\right) + \left(\left(1-\frac{h}{L}\right) - \left[\frac{M_{cr}}{M_a}\right]^{\alpha}\right) I_{cr} \le I_g (3)$$

(C) &ζ are factors, dependent on loading pattern

Ψ is dependent on concrete characteristic strength

$\alpha$is dependent on beam strength and stiffness

The Euro-code[3] used an equation resembling Branson's but with a lower power, as shown in equation (2).

$$I_{\text{eff}} = \left(\frac{M_{cr}}{M_a}\right)^2 I_g + \left(1 - \left[\frac{M_{cr}}{M_a}\right]^2\right) I_{cr} \le I_g \quad (2)$$

It is clear that even international codes did not settle on an equation for evaluating effective moment of inertia. The direct result of this contradiction was enhancing researchers to do more researches. Some researchers adopted artificial intelligence through machine learning, using neural networks asK.A. Patela[4], who expressed the formula shown in equation (3).

$$I_{\text{eff}} = \frac{3I_g}{1+c^{-\left(-7.4688+\frac{8.7116}{1+c^{-H_1}}+\frac{0.3754}{1+c^{-H_2}}+\frac{11.6985}{1+c^{-H_3}}+\frac{10.7167}{1+c^{-H_4}}+\frac{0.6177}{1+c^{-H_5}}+\frac{22.9397}{1+c^{-H_6}}\right)}} \quad (3)$$

Where the factors ($H_{1,2,3,4,5,\&6}$) are tabulatedbased on problem structural parameters.

Preetham et al[5], have also used neural networks to introduce an empirical multiparametric formulation, as shown in equation (4).

$\Delta_{\text{theoretical}}$ = - (0.009*L) - (0.0065*B) - (0.0324*D) + (0.073*F'c) + (0.0222*F_y) - (0.0042*A_{st}) + (0.022*W) + (0.0493*a) - 5.508 (4)Where,

(L) is the span of the beam,(a) is the shear span,(B) is the width of the beam,(D) is the depth of the beam,(A_{st}) is the area of tension steel,(W) is the total load on the beam,(F'c) is the compressive strength of concrete,(F_y)is yield stress of reinforcing steel, and (Δ) is the central deflection of beam

Al-Shaikh and Al-Zaid[6] found that the power of Branson's equation should be presented as a function of reinforcement ratio, as shown in equation (5).

$$I_{\text{eff}} = \left(\frac{M_{cr}}{M_a}\right)^{3-0.8\rho} I_g + \left(1 - \left[\frac{M_{cr}}{M_a}\right]^{3-0.8\rho}\right) I_{cr} \le I_g \quad (5)$$

Where (ρ) is the reinforcement ratio.

MinkwanJu et al[7] studied beams reinforced by special reinforcement types (GFRP) and adopted Branson's equation after modifying the power function and adding a nonlinear parameter (k), as shown in equation (6).

$$I_{\text{eff}} = \left(\frac{M_{cr}}{M_a}\right)^m I_g + \left(1 - \left[\frac{M_{cr}}{M_a}\right]^m - k\right) I_{cr} \le I_g \quad (6)$$

Where $m = 6 - 13\rho_f\frac{E_f}{E_s}$ $k = \frac{1}{11}\left(\frac{M_{cr}}{M_a}\right)^4$

Lian Duan[8] used some different approach as E_cI_{eff} could be calculated from the rotation of the concrete section relative to the applied bending moment as shown in equations (7) up to (11).

$$E_cI_{eff} = \frac{M_n}{\alpha\Phi_y} \quad (7)$$

$$M_n = 0.875Rb\xi d^2\left(1-\frac{\xi}{2}\right) + f_y'A_s'(d-d') \quad (8)$$

$$\xi = \frac{f_yA_s - f_y'A_s'}{0.875Rbd} \quad (9)$$

$$\alpha = 0.75 + 0.5\frac{M}{M_n} \quad (10)$$

$$\Phi d = (0.7 + 2.8\xi)(10)^{-3} + \frac{f_y}{E_s} \quad (11)$$

R. Ian Gilbert[9] exclaimed about why we fail in calculating deflection accurately. He stated that the procedure for calculating deflection proposed by ACI 318[10] is adequate for investigating beams with reinforcement ratio beyond 1.4 %, but is inadequate for lighter reinforcement ratios as the effect of cracking is not adequately encountered for the loss of stiffness.

Amin Ghali et al[11]studied deflection of concrete members of any concrete strength. They studied modifying the conventional method for calculating long term deflection by multiplying the instantaneous deflection by a magnification factor specially for pre-stressed reinforced concrete beams. They proposed a model for calculating deflection based on beam curvature. They stated that for more accurate analysis curvatures at mid and end of beams should be considered. They also proposed factors to calculate deflections away from the mid span. They presented equation (12) for calculating curvature.

$$\Psi_m = (1-\xi)\Psi_1 + \xi\Psi_2 \quad (12)$$

Where

Ψ_m is the curvature at a section between the gross and the cracked ones.

Ψ_1 is the curvature of the gross section.

Ψ_2 is the curvature of the cracked section.

$$\xi = 1\text{-}\beta\left(\frac{M_{cr}}{M}\right)^2 \quad (13)$$

Bischoff[12] performed an analytical study to reach a formula that overcomes Branson's misestimation of deflection of lightly reinforced concrete beams. Even Bischoff used mathematical derivation as a tool, but some assumptions need reconsideration. Equation (14) has been presented by Bishoff, to evaluate effective moment of inertia.

$$I_{eff} = \frac{I_{cr}}{1-\eta\left(\frac{M_{cr}}{M}\right)^2} \quad (14)$$

2. Proposed Formulation

The proposed formulation could be considered a semi-empirical formulation. It has been prepared, based on a modification in the derivation presented by Bishoff[12], considering correlation with experimental data. Figure (2) shows effect of tension stiffening on moment-curvature curve of reinforced beam, where the parameter (β_c) requires to be evaluated.

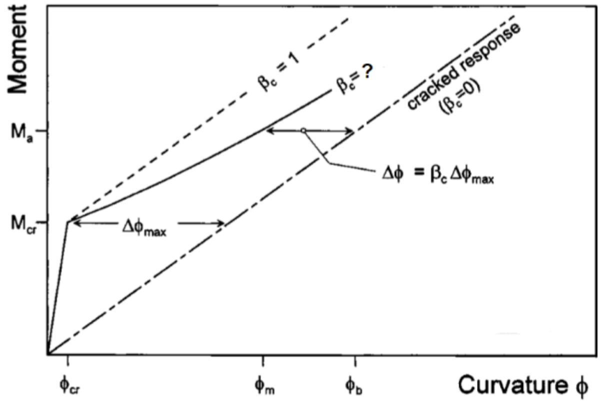


Figure (2) moment-curvature, considering tension stiffening

Presenting tension stiffening, as difference between curvature evaluated based on cracked inertia and that based on gross inertia, results in formulating equation (15) for maximum tension stiffening $\Delta\Phi_{max}$

$$\Delta\Phi_{max} = \frac{M_{cr}}{E_c I_{cr}} - \frac{M_{cr}}{E_c I_g} = \frac{M_{cr}}{E_c I_{cr}}\left(1 - \frac{I_{cr}}{I_g}\right) \quad (15)$$

A factor relating tension stiffening curvature at applied moment to that at cracking (β_c) has to be introduced. The value (β_c) could be empirically evaluated based on experimental results, putting into consideration that it should be a function of the applied moment and cracking moment.

$$\Phi_m = \Phi_b - \Delta\Phi = \frac{M_a}{E_c I_{cr}} - \beta_c \Delta\Phi_{max} \quad (16)$$

$$\Phi_m = \frac{M_a}{E_c I_{cr}} - \beta_c \frac{M_{cr}}{E_c I_{cr}}\left(1 - \frac{I_{cr}}{I_g}\right) \quad (17)$$

$$\Phi_m = \frac{M_a}{E_c I_{cr}}\left(1 - \beta_c \frac{M_{cr}}{M_a}\left(1 - \frac{I_{cr}}{I_g}\right)\right) \quad (18)$$

$$EI_{eff} = \frac{M_a}{\Phi_m} = \frac{E_c I_{cr}}{1 - \beta_c \frac{M_{cr}}{M_a}\left(1 - \frac{I_{cr}}{I_g}\right)} \quad (19)$$

And since concrete modulus of elasticity is a material property, defined by initial tangent modulus or secant modulus (independent of state of stresses); then.

$$I_{eff} = \frac{I_{cr}}{1 - \beta_c \frac{M_{cr}}{M_a}\left(1 - \frac{I_{cr}}{I_g}\right)} \quad (20)$$

(β_c) has been evaluated through the experimental program and it has beenpresented via equation (21)

$$\beta_c = \left(\frac{M_{cr}}{M_a}\right)^{\left(\frac{10 A_s}{A_{sb}}\right) - 1} \quad (21)$$

Where (A_{sb}) is the balanced reinforcement ratio

Substituting in equation (20), the proposed formula for effective inertia could be presented via equation (22)

$$I_{eff} = \frac{I_{cr}}{1 - \left(1 - \frac{I_{cr}}{I_g}\right)\left(\frac{M_{cr}}{M_a}\right)^{\frac{10 A_s}{A_{sb}} - 1}} \quad (22)$$

3. Experimental Program

An experimental program has been conducted to evaluate significance of different parameters on evaluating the power of effective inertia equation. Ten specimens have been examined in four points bend configuration; to investigate roles of concrete characteristic strength, steel yield stress, reinforcement ratio, and slenderness ratio. Table (1) summarizes structural data of test specimens, while figure (3) shows specimen's configuration, and figure (4) shows test setup.

Table (1) Details of the experimental specimens.

SPECIMEN	RFT	DIM mm	F_Y MPa	F_{CU} MPa
32N-M54	3ϕ10	150x320	240	31.8
32N-H54	3ϕ10	150x320	400	33.9
32N-M78	3ϕ12	150x320	240	33.5
32N-H78	3ϕ12	150x320	400	32.5
32N-M14	3ϕ16	150x320	240	32.9
32N-H14	3ϕ16	150x320	400	34.1
20N-M78	2ϕ10	120X200	240	32.7
20N-H78	2ϕ10	120X200	400	30.8
20H-M78	2ϕ10	120X200	240	60.5
20H-H78	2ϕ10	120X200	400	58.6

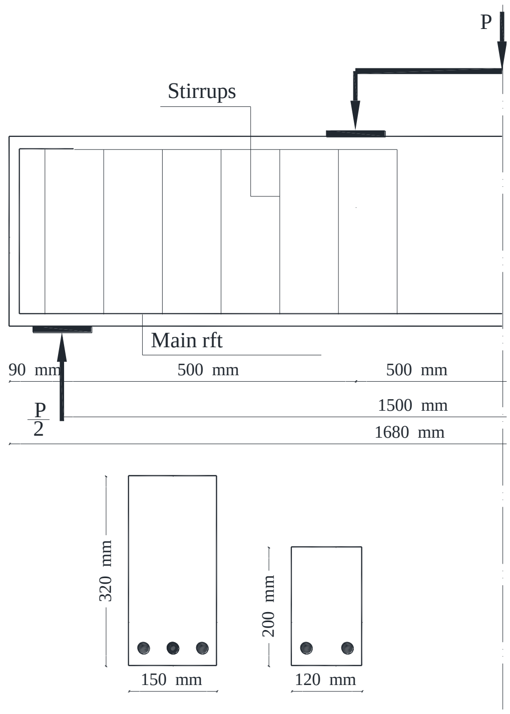


Figure (3) Specimen configuration

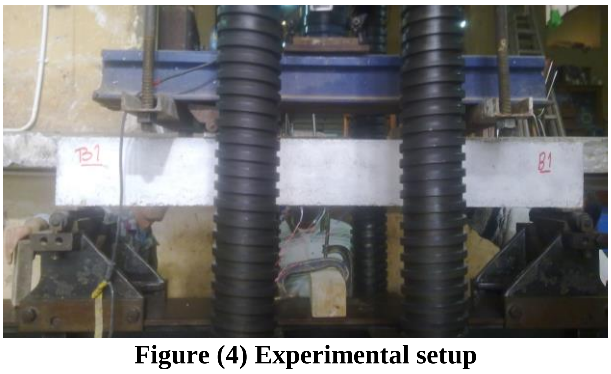


Figure (4) Experimental setup

Strain gauges have been installed on tensile steel reinforcement, and compressed concrete surface. while LVDTs were installed to measure mid-span deflection. Crack pattern has also been traced across beams surfaces.

4. Analysis and discussion

Analysis of experimental outputs has been performed in order to reach valuable conclusions, regarding research point. Crack pattern has first been studied as it is the most obvious indicator for beam's structural behavior. Crack pattern has been traced on beam's surfaces during testing and load levels have been plotted on cracks. All beams have been designed to fail in flexure, even shear crack did not propagate to form shear failure. Figures (5) up to (14) present details of crack pattern tracing for all tested specimens.

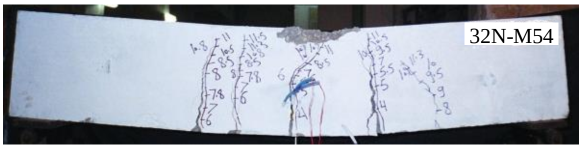


Figure (5) Crack pattern of [32N-M54]

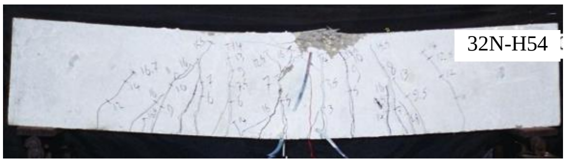


Figure (6) Crack pattern of [32N-H54]

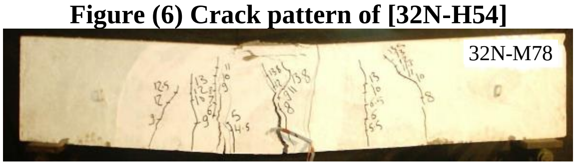


Figure (7) Crack pattern of [32N-M78]

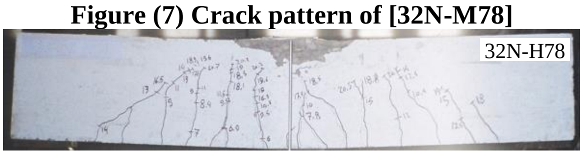


Figure (8) Crack pattern of [32N-H78]

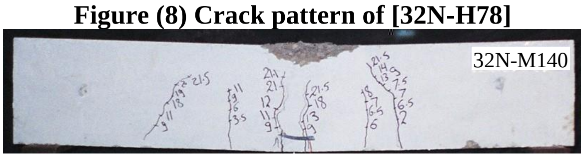


Figure (9) Crack pattern of [32N-M140]

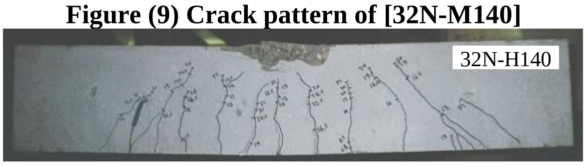


Figure (10) Crack pattern of [32N-H140]

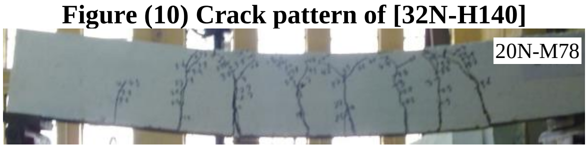


Figure (11) Crack pattern of [20N-M78]

Figure (12) Crack pattern of [20N-H78]

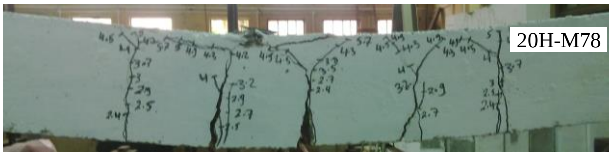


Figure (13) Crack pattern of [20H-M78]

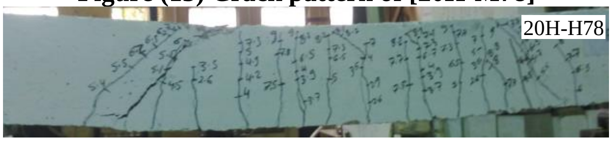


Figure (14) Crack pattern of [20H-H78]

The study of crack pattern form showed that the increase in yield stress resulted in more cracking associated with small crack width. This could be attributed to two factors. The first is that high grade reinforcement was deformed while mild was smooth (market constraint). The second factor is that higher grade reinforcement allows increase in applied load before steel reached yield plateau, which enables additional cracks to form before existing cracks widths increase extensively due to yielding.

It could also be noticed that cracks of high strength specimens propagate in a more linear pattern. This refers to the increased strength of cement paste, that results in passing cracks through aggregate rather zigzagging around aggregate-mortar interfaces.

Strain growth in both concrete compressed fibers and main reinforcing steel has been plotted in figures (15) up to (19), for samples of tested specimens to check functioning of test configuration.

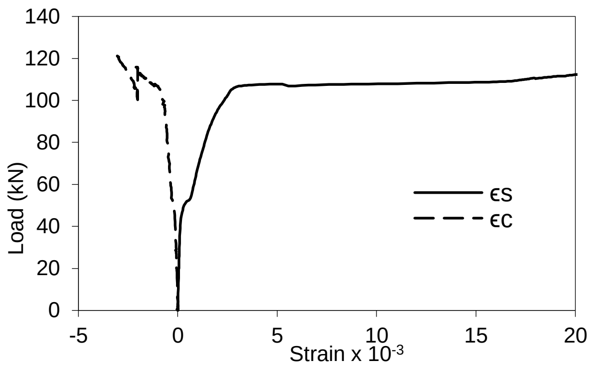


Figure (15) Strain of [32N-M54]

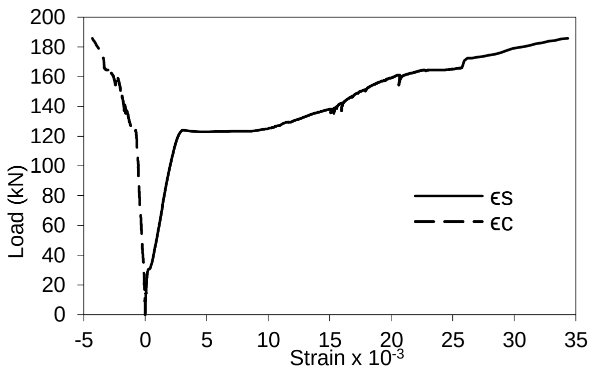


Figure (16) Strain of [32N-H54]

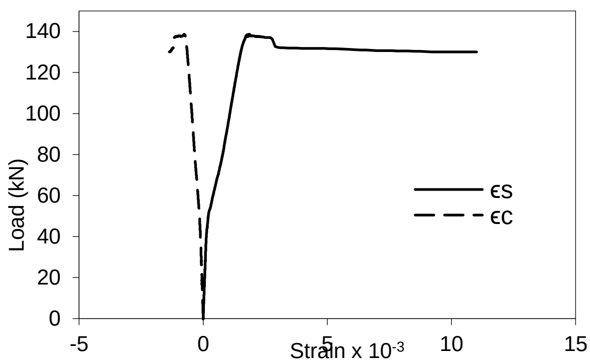


Figure (17) Strain of [32N-M78]

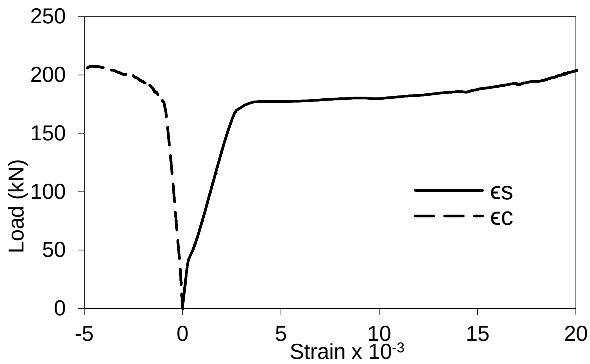


Figure (18) Strain of [32N-H78]

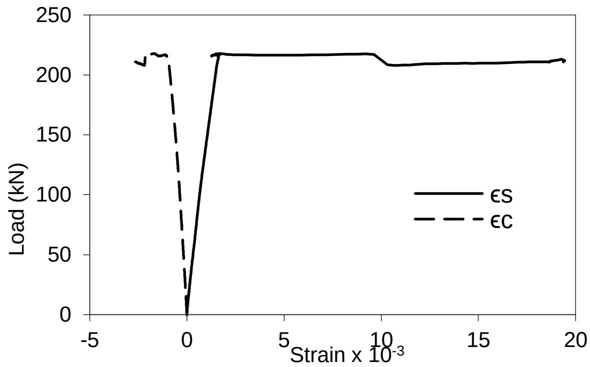


Figure (19) Strain of [32N-M140]

It could be seen that steel maximum tensile passes by three different stages. The first is of maximum load-strain slope before cracking begins. The second stage begins after the formation of the first crack, where load-strain slope decreases and consequently strain increases at a higher rate. It could be noticed that the decrease in curve slope is higher for sections of low reinforcement ratio, where number of cracks is limited and crack widths are wide. The third stage is yielding, where strains grow extensively at the same loading level. Concrete maximum strain was limited below 0.005, before which crushing failure took place.

Since deflection is the main concern of this paper it has been plotted for all beams as shown in figures (20) up to (29). The proposed formulation has been plotted in comparison with experimental and other formulations available in the literature.

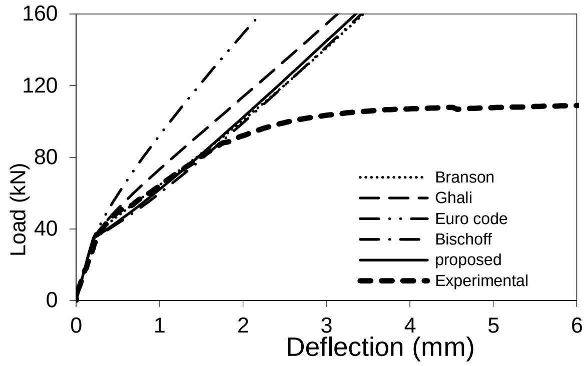


Figure (20) Deflection of [32N-M54]

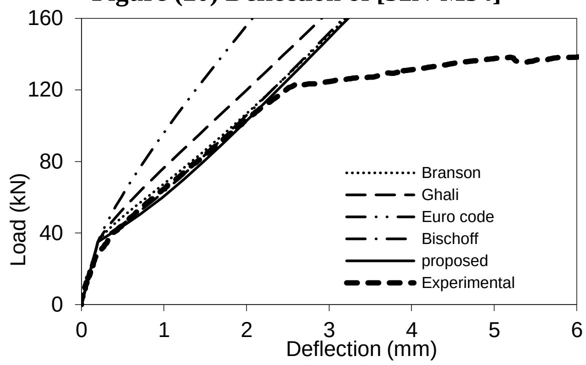


Figure (21) Deflection of [32N-H54]

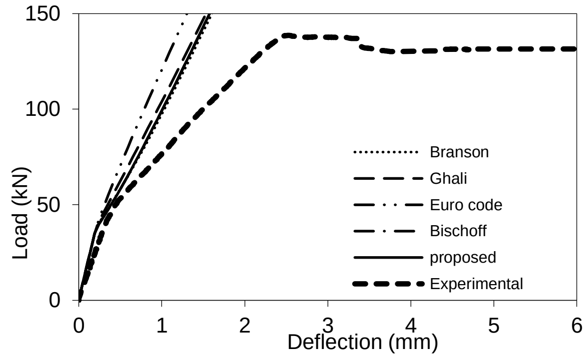


Figure (22) Deflection of [32N-M78]

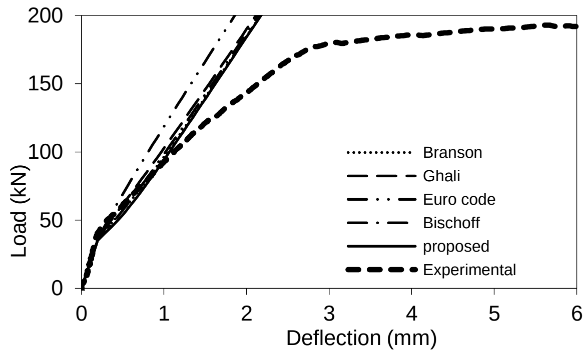


Figure (23) Deflection of [32N-H78]

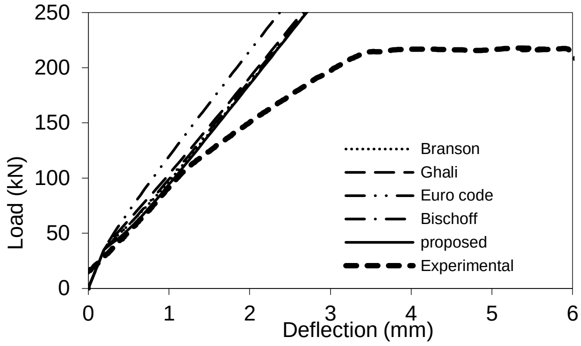


Figure (24) Deflection of [32N-M140]

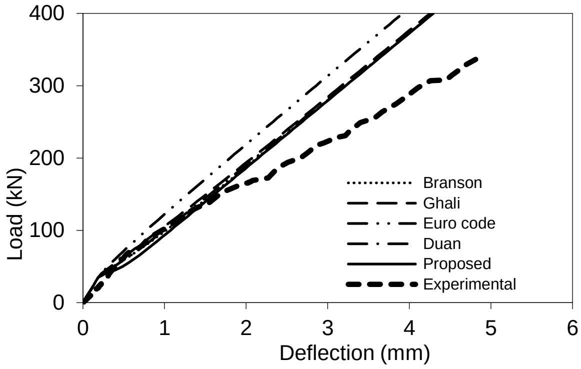


Figure (25) Deflection of [32N-H140]

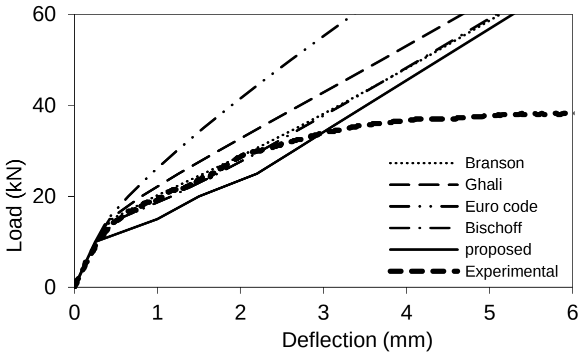


Figure (26) Deflection of [20N-M78]

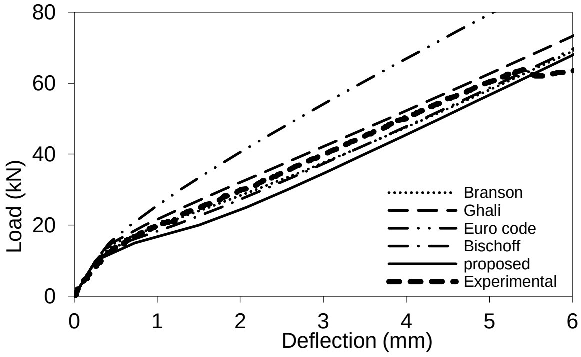


Figure (27) Deflection of [20N-H78]

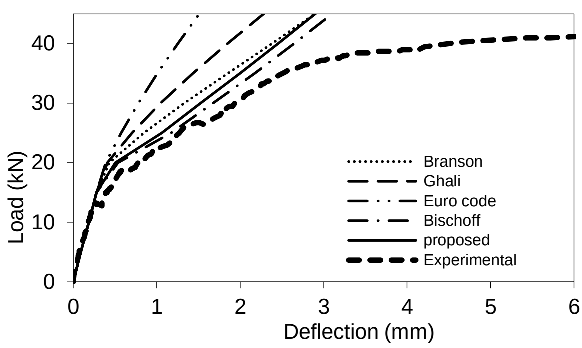


Figure (28) Deflection of [20H-M78]

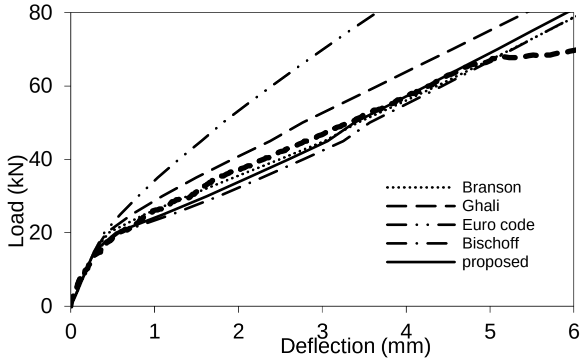
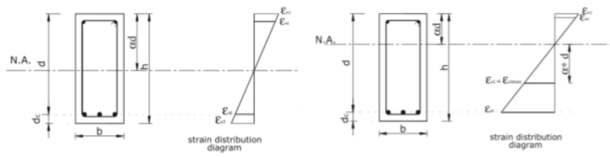


Figure (29) Deflection of [20H-H78]

The Study of deflection figures showed that the proposed model has proven a high correlation with experimental data for most specimens than other models available in the literature.

It could be seen that deflection of all specimen consisted of two regions (before and after cracking). Loads at which deflection behavior changes (change in P-Δ slope) have been compatible with crack patterns and load strain curves, presented before.

The comparison performed between specimens of different reinforcement ratios showed that tension stiffening decreases by the increase in reinforcement ratio. Figure (30) clarifies this idea, as it shows strain distribution in cracked and un-cracked sections. It could be concluded that inertia of cracked section is proportional to reinforcement ratio, and consequently difference between cracked and uncracked inertia decreases by the increase in reinforcement ratio.



a) Uncracked section b) Cracked section

Figure (30) Strain distribution in concrete sections subjected to flexure

Effect of beam stiffness has been investigated by comparing beams of the same reinforcement ratio (0.78%) and different stiffness (32N-M78 with 20N-M78 and 32N-H78 with 20N-H78). it could be seen that tension stiffening increases by the decrease in beam stiffness. This refers to excessive cracking that took place in beams of low stiffness than those of high stiffness, as shown in crack pattern figures.

The role of concrete characteristic strength has been investigated by comparing specimens (20N-M78 with 20H-M78 and 20N-H78 with 20H-H78).Even the increase in concrete characteristic strength should result in an increase in tension stiffening, due to the decrease in cracked section inertia; but this could not be noticed clearly in load deflection curves; due to the counter effect of increased modulus of elasticity, resulting from increased characteristic strength

5. Conclusion

Both the experimental program and the mathematical formulation have been used to reach a modified formula for the effective moment of inertia for reinforced concrete beams subjected to flexure. The proposed formulation could be considered a modification to Bischoff formula, considering non-linearity of tension stiffening coefficient via experimental calibration. The proposed formulation showed an accepted correlation with experimental results.

Comparison between various effective moment of inertia formulae has been performed. It was found that the Euro-code formula overestimates effective moment of inertia for most tested specimens. Bischoff and Branson's formulae showed accepted correlation with specimens of high reinforcement ratios.

Effect of tension stiffening was found to be non-linearly proportional to the increase in the applied moment after cracking.

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