Limit state of anisotropic rock massif near a mine opening

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Abstract. We consider approximated solution of elasticoplastic problem close to a cavity in laminated (layered) massif with the assumptions that in elastic zone the massif is anisotropic and obeys the generalized Hooke's law, and the inelastic zone is modeled as an isotropic medium. The problem is approximately solved using the P.I.Perlin's method involving an iterative scheme. A system of algebraic equations has been created for finding unknown coefficients of complex potential. For the same elastic parameters of anisotropy, comparison is shown of dimensions of the inelastic deformation zone for various conditions of plasticity near a mining opening of the same depth.
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Introduction

Influence of internal pressure at opening contour on the size and configuration of plastic zones close to mine opening is numerically shown. An algorithm has been made for digital implementation of the indicated problem using a PC, and results are shown as graphs.

Design and construction of underground facilities requires a justified approach to assessing the impact of static and seismic loads on various types of structural elements of buildings, their definition of stress and stressed state basing on improving elasticoplastic models.

The method of elasticoplastic problem algebraization was used by R. Nottrot [1] and R. Timman [2] for obtaining a numerical solution to some specific problems.

In case of sufficiently high level of static and dynamic loads, rock massif around mine openings may come to its limit state and the values of static and seismic stresses may exceed the ultimate strength of rock massif, leading to formation of inelastic deformation areas.

The elasticoplastic problem of straining a band with semicircular cuts was solved by R. Southwell and R. Allen using the relaxation method [3]

The problem was solved by using the P.N.Perlin's semi-inverse method [4, 5] and the algorithm shown in works [6-8].

Distribution of elastic stresses components in layered medium close to the cavity

In accordance with initial presupposition, rock massif in an elastic zone obeys the generalized Hooke's law for a transtropic body with horizontal isotropy plane, which for tunnels in plane deformation conditions are written in as [9]:

$$\varepsilon_x = b_{11}\sigma_x + b_{12}\sigma_y,$$
$$\varepsilon_y = b_{12}\sigma_x + b_{22}\sigma_y, \tag{1}$$
$$\gamma_{xy} = b_{66}\tau_{xy}$$

Here "y" above indicates components of stresses: Deformations in elastic zone. Coefficients of deformation b_{ij}, (i, j=1.2, 6), are equal to:

$$b_{11} = \frac{1-v_1^2}{E_1},\ b_{22} = \frac{1}{E_2} - \frac{v_2^2}{E_1},$$
$$b_{12} = \frac{v_2(1+v_1)}{E_1},\ b_{66} = \frac{1}{G_2} \tag{2}$$

where E_1, E_2 are Young modulus for compression straining in the direction of isotropy plane and normal to it; v_1 is Poisson's ratio characterizing lateral contraction in the plane of isotropy in case of straining in this plane: v_2 same as in case of straining in the direction perpendicular to the isotropy plane; G_2 is the shear modulus for the plane perpendicular to the isotropy plane.

Now let us the problem of an anisotropic body elasticity theory for an infinite plane with an elliptic opening, on the circuit of which normal and tangential stresses are applied symmetric to the coordinate axes, and infinities are exposed to strains:

$$\sigma_x^{(\infty)} = -p,\ \ \sigma_y^{(\infty)} = -q,\ \ \tau_{xy}^{(\infty)} = 0 \tag{3}$$

Stresses in the elastic zone, as it is known [10], are presented in two sophisticated analytic functions $\varphi_k(z_k)$ of a complex argument

$z_k = x + \mu_k y$, ($\kappa=1.2$):

$$\sigma_x^e = 2\,\text{Re}\left[\mu_1^2 \varphi_1'(z_1) + \mu_2^2 \varphi_2'(z_2)\right],$$

$$\sigma_y^e = 2\,\text{Re}\left[\mu_1' (z_1) + \varphi_2'(z_2)\right],$$

$$\tau_{xy}^e = -2\,\text{Re}\left[\mu_1 \varphi_1'(z_1) + \mu_2 \varphi_2'(z_2)\right].$$
(4)

Here μ_k is found as a root of the characteristic equation of the fourth degree. [9]

$$\mu^4 + a_1 \mu^2 + a_2 = 0,$$
(5)

where $a_1 = \dfrac{2b_{12} + b_{66}}{b_{11}}$, $a_2 = \dfrac{b_{22}}{b_{11}}$

For many anisotropic bodies μ_k are purely imaginary values, i.e. $\mu_k = i\beta_k$.

Values β_k that define the degree of anisotropy of a body are called elastic parameters of anisotropy; for an isotropic body $\beta_k = 1$, (k=1.2). These parameters are determined by five elastic constants E_k, V_K, G_2. (k=1.2), by the following formulas:

$$\left.\begin{array}{c}\beta_1 \\ \beta_2\end{array}\right\} = \frac{1}{2}\left[\sqrt{\frac{2b_{12}+b_{66}}{b_{11}} + \sqrt{\frac{b_{22}}{b_{11}}}} \pm \sqrt{\frac{2b_{12}+b_{66}}{b_{11}} - 2\sqrt{\frac{b_{22}}{b_{11}}}}\right] = \frac{1}{2}\left[n \pm \sqrt{m-2k}\right]$$
(6)

Here

$$m = \frac{l_1 - 2v_2(1+v_1)}{1-v_1^2}, \quad k = \sqrt{\frac{1-v_2^2}{1-v_1^2}},$$

$$n = \sqrt{m+2k}, \quad l = \frac{E_1}{E_2}, \quad l_1 = \frac{E_1}{G_2}.$$

Values n, k characterize the measure of the anisotropy of the body in case of plane deformation and show deviation from the isotropic body, for which n = 1, k= 2.

Let us show strain functions $\varphi_k(z_k)$, $(k = 1, 2)$, as [10].

$$\varphi_k(z_k) = \varphi_k^{(0)}(z_k) + \varphi_k^{(00)}(z_k) = A_{k0}z_k + \varphi_k^{(00)}(z_k)$$
(7)

Functions of the main strains in the untouched massif $\varphi_k^{(0)}(z_k)$, i.e, A_{k0} constants are associated with the strains at the infinity and elastic parameters β_k (k = 1.2) depending on:

$$A_{10} = -\frac{\sigma_x^{(\infty)} + \beta_2^2 \sigma_y^{(\infty)}}{2(\beta_1^2 - \beta_2^2)}, \quad A_{20} = \frac{\sigma_x^{(\infty)} + \beta_1^2 \sigma_y^{(\infty)}}{2(\beta_1^2 - \beta_2^2)}.$$
(8)

To find additional strain functions $\varphi_k^{(00)}(z_k)$, (k = 1.2) related to mining, let us display appearance of an elliptic contour with axes OA = a,

OB = b onto the appearance of a unit circle, using a rational function $\omega(\zeta)$ in form of

$$z = \omega(\zeta) = m_1\left(\zeta + \frac{m_2}{\zeta}\right),$$
(9)

Then $z_k = \omega_k(\zeta_k) = \dfrac{a+\beta_k b}{2}\zeta_k + \dfrac{a - \beta_k b}{2}\cdot\dfrac{1}{\zeta_k}$,
10)

where

$$\zeta_k = \frac{z_k + \sqrt{z_k^2 - sqr(a) + \beta_k^2 b^2}}{a + \beta_k b};$$

on the contour of the unit circle $\zeta_k = \sigma = e^{i\theta}$.

By denoting the infinite plane with an elliptical opening by S, we can argue that the functions $\varphi_k(z_k)$ are defined in S_k areas obtained from S by affine transformation.

The following expressions are used as boundary conditions for defining functions $\varphi_k^{(00)}(z_k)$,

$$2\,\text{Re}\left[\varphi_1^{(00)}(z_1) + \varphi_2^{(00)}(z_2)\right] = -\int_0^s Y_n dS,$$
(11)

$$2\,\text{Re}\left[i\beta_1 \varphi_1^{(00)}(z_1) + i\beta_2 \varphi_2^{(00)}(z_2)\right] = \int_0^s X_n dS.$$

In the last relations X_n and Y_n, projections of external forces on contour of elliptical opening to the corresponding coordinate axes.

Knowing that on the contour of the opening

$$\varphi_1^{(00)}(z_1) = \varphi_1^{(00)}[\omega_1(\sigma)] = \omega_{10}(\sigma),$$

$$\varphi_2^{(00)}(z_2) = \varphi_2^{(00)}[\omega_2(\sigma)] = \omega_{20}(\sigma),$$

and expanding the right-hand sides of boundary conditions (11) into a Fourier series, let us reduce them to:

$$2\,\text{Re}[\varphi_{10}(\sigma) + \varphi_{20}(\sigma)] = \sum_{n=1}^{\infty}\left(a_n \sigma^n + \overline{a_n \sigma^{-n}}\right),$$
(12)

$$2\,\text{Re}[i\beta_1 \varphi_1(\sigma) + i\beta_2 \varphi_2(\sigma)] = \sum_{n=1}^{\infty}\left(b_n \sigma^n + \overline{b_n \sigma^{-n}}\right)$$

Due to symmetry of the problem (with respect to the coordinate axes), summation in the right parts of (12) is performed using odd powers only. σ

Using known properties of the integral of Cauchy type and going to old variables Z_k, from boundary conditions (12) one can find functions $\varphi_k^{(00)}(z_k)$ in the following form:

$$\varphi_k^{(00)}(z_k) = \sum_{n=1}^{\infty} A_{kn}\left[\frac{a+\beta_k b}{z_k + \sqrt{z_k^2 - a^2 + \beta_k^2 b^2}}\right]^{2n-1}$$
(13)

Where coefficients A_{kn}, ($k=$ 1.2,...), unknown actual values.

Thus, full functions $\varphi_k(z_k)$ of strains are written as

$$\varphi_k(z_k) = A_{k0}z_k + \sum_{n=1}^{\infty} A_{kn}\left[\frac{a+\beta_k b}{z_k + \sqrt{z_k^2 - a^2 + \beta_k^2 b^2}}\right]^{2n-1} \quad (14)$$

Derivatives of these functions $\varphi_k'(z_k)$ are equal to:

$$\varphi_k'(z_k) = A_{k0} - \frac{1}{\sqrt{z_k^2 - a^2 + \beta_k^2 b^2}}\sum_{n=1}^{\infty} A_{kn}\left[\frac{(2n-1)(a+\beta_k b)}{z_k + \sqrt{z_k^2 - a^2 + \beta_k^2 b^2}}\right]^{2n-1} \quad (15)$$

Finally strain components in an elastic zone through functions (15) are defined by expressions:

$$\sigma_x^e = -\left\{2\left(A_{10}\beta_1^2 + A_{20}\beta_2^2\right) + 2\operatorname{Re}\sum_{k=1}^{2}\beta_k^2\frac{1}{\sqrt{z_k^2 - a^2 + \beta_k^2 b^2}}\times\right.$$

$$\left.\times\sum_{n=1}^{\infty}(2n-1)A_{kn}\left(\frac{a+\beta_k b}{z_k + \sqrt{z_k^2 - a^2 + \beta_k^2 b^2}}\right)^{2n-1}\right\},$$

$$\sigma_y^e = \left\{2\left(A_{10} + A_{20}\right) - 2\operatorname{Re}\sum_{k=1}^{2}\beta_k^2\frac{1}{\sqrt{z_k^2 - a^2 + \beta_k^2 b^2}}\times\right.$$

$$\left.\times\sum_{n=1}^{\infty}(2n-1)A_{kn}\left(\frac{a+\beta_k b}{z_k + \sqrt{z_k^2 - a^2 + \beta_k^2 b^2}}\right)^{2n-1}\right\},$$

$$(16)$$

$$\tau_{xy}^e = -2\operatorname{Re}\sum_{k=1}^{2}i\beta_k\sum_{n=1}^{\infty}\frac{(2n-1)A_{kn}}{\sqrt{z_k^2 - a^2 + \beta_k^2 b^2}}\left(\frac{a+\beta_k b}{z_k + \sqrt{z_k^2 - a^2 + \beta_k^2 b^2}}\right)^{2n-1}$$

Distribution of strains in zone of plasticity in case of Hoek-Brown instability

Due to statistical determinability of the problem, strain components in the plastic zone exist without including elasticoplastic boundary, and depend only on the boundary conditions at the contour of the cavity. Let's proceed to defining them.

Around a circular mine opening, it is convenient to show strain components in polar coordinates. Let's indicate them by the "n" index above, indicating that they belong to the plastic zone. Stress components σ_r^p, σ_θ^p, $\tau_{r\theta}^p$ in the plastic zone satisfy differential equations of equilibrium.

$$\frac{\partial \sigma_r^p}{\partial r} + \frac{1}{r}\frac{\partial r_{r\theta}^p}{\partial \theta} + \frac{\sigma_r^p - \sigma_\theta^p}{r} = 0,$$

$$\frac{\partial \tau_{r\theta}^p}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta^p}{\partial \theta} + \frac{2\tau_{r\theta}^p}{r} = 0 \quad (17)$$

for boundary conditions on the contour of the cavity (at $r = 1$)

$$\sigma_r^p = p_0 = const, \tau_{r\theta}^p = 0. \quad (18)$$

and Hoek-Brown plasticity condition [11]

$$\left(\sigma_r^p - \sigma_\theta^p\right) - \sqrt{-m\sigma_r^p\sigma_c + s\sigma_c^2} = 0, (19)$$

where $\sigma_c > 0,$ is resistance to simple compression of intact rock, the values are taken from the experiment; s is the parameter (value) defining the level of crackling (1 stands for absence of damage and 0 (zero) for completely broken material).

The following assumptions are valid:

a) the area of inelastic deformation completely covers unattached contour of mine opening radius R;

b) the isotropic incompressible material in the area of inelastic deformation obeys criterion of Hoek-Brown instability without mitigation;

c) the elastic area is under plain strain and its behavior is described by generalized Hooke's law for homogeneous transtropic massif with inclined isotropy plane.

Next, let's define expression of various fields within the plastic area that are completely covered by the mining opening contour with circular cross section. Do to so, let's use the fact that the plasticity criterion is achieved throughout the limit area, which allows to record σ_θ as σ_r and solve the equilibrium equation. The resulting differential equation:

$$\frac{\sigma_r^p}{r} = -\frac{\sqrt{-m\sigma_c\sigma_r - s\sigma_c^2}}{r} \quad (20)$$

with boundary conditions r=R, $\sigma_r^p = -P_i$

where P_i is internal pressure, m is the parameter associated with rock massif properties (usually 5 to 30), where the letter p above have components of plastic strains.

Converting differential root of a complex functions:

$$\frac{d\sigma_r^p}{\sqrt{-m\sigma_c\sigma_r + s\sigma_c^2}} = -\frac{dr}{r}. \qquad (21)$$

Thus, components of plastic strains in the areas of a polar coordinate system are the following:

$$\sigma_r^p = \frac{s\sigma_c}{m} - \frac{1}{m\sigma_c}\left(\sqrt{s\sigma_c^2 - m\sigma_c P_i} + \frac{m\sigma_c}{2}\ln\frac{r}{R}\right)^2 \qquad (22)$$

$$\sigma_\theta^p = \sigma_r^p - \sqrt{s\sigma_c^2 - m\sigma_c\sigma_r^p}.$$

In the original variant, let's that internal pressure is zero (P=0). $_i$

Due to static determinability of the problem in a plastic area, strains components in rectangular coordinates are found independently of strains at the "infinity" by formulas:

$$-\sigma_x^p/\sigma_c = \Psi\frac{(z+\bar{z})^2}{4z\bar{z}} + \left(\Psi + \sqrt{1+m\Psi}\right)\frac{(z-\bar{z})^2}{4z\bar{z}},$$

$$-\sigma_y^p/\sigma_c = \Psi\frac{(z-\bar{z})^2}{4z\bar{z}}\left(\Psi + \sqrt{1+m\Psi}\right)\frac{(z+\bar{z})^2}{4z\bar{z}},$$

$$-\tau_{xy}^p/\sigma_c = \left(\Psi + 1 + \sqrt{1+m\Psi}\right)\frac{z^2-\bar{z}^2}{4iz\bar{z}},$$

$$(23)$$

where

$$\Psi = \ln\sqrt{\frac{z\bar{z}}{R^2}}\left(1 + \frac{m}{4}\ln\sqrt{\frac{z\bar{z}}{R^2}}\right), z = x+iy, \bar{z} = x-iy.$$

In the second case, when pressure P_i $(P_i > 0, s \neq 1)$ acts from the internal contour of the mine opening, strain components in the plastic zone are defined as follows:

$$\frac{\sigma_x^p}{\sigma_c} = -\eta^*\frac{(z+\bar{z})^2}{4|z\bar{z}|} - \left(\eta^* + N\right)\frac{(z-\bar{z})^2}{4|z\bar{z}|} + \frac{P_i}{\sigma_c};$$

$$\frac{\varpi_y^p}{\sigma_c} = -\eta^*\frac{(z+\bar{z})^2}{4|z\bar{z}|} - \left(\eta^* + N\right)\frac{(z-\bar{z})^2}{4|z\bar{z}|} + \frac{P_i}{\sigma_c};$$

$$(24)$$

$$\frac{\tau_{xy}^p}{\sigma_c} = N\frac{z^2-\bar{z}^2}{4i|z\bar{z}|}.$$

where

$$\eta^* = \ln\sqrt{\frac{|z\bar{z}|}{R^2}}\left(\sqrt{s - m\frac{P_i}{\sigma_c}}\right) - \frac{m}{4}\ln\sqrt{\frac{|z-\bar{z}|}{R}}, N = \sqrt{s + m\eta^* - m\frac{P_i}{\sigma_c}}, z = x+iy, \bar{z} = x-iy$$

Algorithm for numerical solution of an elasticoplastic problem near a cavity

Now let us directly find unknown coefficients A_{k0} and A_{kn} (k = 1.2; n = 1.2,...) of elastic strain functions (14) and clarify prerequisites of elasticoplastic boundary defined initially in the form of an ellipse.

The strain at the desired boundary is to be continuous:

$$\sigma_x^e = \sigma_x^p, \qquad \sigma_y^e = \sigma_y^p, \qquad \tau_{xy}^e = \tau_{xy}^p. \quad (25)$$

In these conditions, elastic strains are represented by expressions (16) and plastic strains - by equations (23) or (24) for the Hoek-Brown plasticity condition.

In the first quarter of the displayed plane ζ let us put m intermediate directions (beams) at angles $\theta_{\zeta j}$, $(j = 1,2,3,...)$ to axis 0ξ (Fig. 2). The area in question is limited to directions $\theta_{\zeta,0} = 0, \theta_{\zeta,(m+1)} = \pi/2$, that correspond to fixed points A and B in the plane Z. Let's denote radial coordinates (the distance from the pole in a polar system) of points on selected beams as $\rho_{\zeta j}$

According to semi-inverse method [4, 5] positions of any two points in the elasticoplastic boundaries are provided. Let these be the point of intersection between axes with an ellipse, i.e., OA = a, OB = b (see Fig. 1). An ellipse with specified semi-axes we will set as the elasticoplastic boundary at zero approximation. Let's assume that the required elasticoplastic boundary passes these points, and according to the condition that points A and B are fixed in plane z $\rho_{\zeta,0} = \rho_{\zeta,(m+1)} = 1,0$. In other words, agitated are radial coordinates $\rho_{\zeta j}$ close to position $\rho_{\zeta j} = 1,0$ that correspond to the points of the ellipse in plane z, except for the two extreme points with coordinates $\left(\theta_{\zeta,0} = 0, \rho_{\zeta,0} = 1,0\right)$ and

$$\left(\theta_{\zeta,(m+1)} = \pi / 2, \ \rho_{\zeta(m+1)} = 1,0 \right).$$
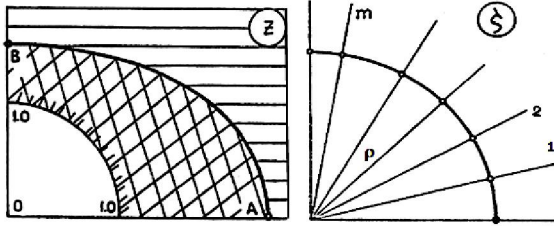

Figure 1. Design scheme defining elasticoplastic boundary around the cavity in an anisotropic body: a) plastic zone around the cavity; b) area with a unity circle.

Let's limit the upper limit of infinite sums in (14) by some number N *and write the conditions (25) for each of the selected* $(m+2)$ *points.* Then, to determine 2(N+1) unknown coefficients A_{k0}, A_{kn}, (K = 1,2; n = 1,2,..., N) and m unknown $\rho_{\zeta j} \ (j = 1, 2, ..., m)$ we get a system $(3m+4)$ of heterogeneous algebraic equations.

Let us note that due to the fact that in case of $\theta_{\zeta,0} = 0$ and $\theta_{\zeta(m+1)} = \pi / 2, \ \tau_{xy}^{\Pi} = 0$ and $\tau_{xy}^{y} \equiv 0$, two equations fall from conditions (4.25) as identities). The resulting system of equations is linear in relation of coefficients A_{k0}, A_{kn} and is highly nonlinear in relation to $\rho_{\zeta j}$.

From the condition of equality of number in equations (3m+4) and the number of unknown 2(N+1)+m we can find the upper limit of the sum in (4.14), i.e., N=m+1. Thus, the order of the system of equations depends on the number of selected intermediate points on elasticoplastic boundary.

Let us reduce equation (4) to the form suitable for later use:

$$\varphi_1'(Z_1) + \overline{\varphi_1'(Z_1)} = \frac{\sigma_x^e + \beta_2^2 \, \sigma_y^e}{2\left(\beta_1^2 - \beta_2^2\right)}$$

$$\varphi_2'(Z_2) + \overline{\varphi_2'(Z_2)} = \frac{\sigma_x^e + \beta_1^2 \, \sigma_y^e}{2\left(\beta_1^2 - \beta_2^2\right)} \qquad (26)$$

$$i\beta_1 \varphi_1'(Z_1) - i\beta_1 \overline{\varphi_1'(Z_1)} + i\beta_2 \varphi_2'(Z_2) - i\beta_2 \overline{\varphi_2'(Z_2)} = -\tau_{xy}^e.$$

Let's insert expression (15) into the first two equations (26) instead of the derivatives of strain functions. Then, taking into account the first two conditions (25) for each of (m+2) directions, we obtain a system of algebraic equations of order 2(m+2) to determine 2(m+2) coefficients A_{k0}, $A_{kn} \ (K = 1, 2; \ n = 1, 2, ..., m + 1):$

$$A_{10} - \mathrm{Re} \sum_{n=1}^{m+1} \frac{(2n-1) A_{1n}}{\sqrt{Z_{1j}^2 - a^2 + \beta_1^2 b^2}} \left(\frac{a + \beta_1 b}{Z_{1j} + \sqrt{Z_{1j}^2 - a^2 + \beta_1^2 b^2}} \right)^{2n-1} =$$

$$-\frac{\sigma_{x_j}^p + \beta_2^2 \sigma_{y_j}^p}{2\left(\beta_1^2 - \beta_2^2\right)}, \qquad (27)$$

$$A_{20} - \mathrm{Re} \sum_{n=1}^{m+1} \frac{(2n-1) A_{2n}}{\sqrt{Z_{1j}^2 - a^2 + \beta_2^2 b^2}} \left(\frac{a + \beta_2 b}{Z_{1j} + \sqrt{Z_{2j}^2 - a^2 + \beta_2^2 b^2}} \right)^{2n-1} =$$

$$-\frac{\sigma_{x_j}^p + \beta_1^2 \sigma_{y_j}^p}{2\left(\beta_1^2 - \beta_2^2\right)},$$

Here

$$Z_{kj} = x_j + i\beta_k \, y_i, \ (K = 1, 2; \ j = 1, 2, ..., m + 1);$$

$$x_j = \frac{a+b}{2}\left(\rho_j + \frac{a-b}{a+b} \cdot \frac{1}{\rho_j} \right)\cos\theta_j, \ y_j = \frac{a+b}{2}\left(\rho_j + \frac{a-b}{a+b} \cdot \frac{1}{\rho_j} \right)\sin\theta_j$$

(28)

Note that taking into account the previously set value N=m+1, we have the number of unknown 2 (m+2)=2(N+1).

Proceeding similarly, from the third equation (26) taking into account the last equation (25), we obtain a system of m equations:

$$\mathrm{Re} \sum_{K=1}^{2} \frac{i\beta k}{\sqrt{Z_{K1}^2 - a^2 + \beta_k^2 b^2}} \sum_{n=1}^{m+1} A_{KI}(2n-1)\left(\frac{a+\beta_k b}{Z_{K1}\sqrt{Z_{K1}^2 - a^2 + \beta_k^2 b^2}} \right)^{2n-1} = -0.5\tau_{x_{0i}}^n \ (i = 1.2,...m).$$

(29)

These m equations play the role of reference ones for assessing accuracy of the strain continuity conditions when passing through an elasticoplastic boundary for a given set of m values $\rho_{\zeta j}, (j = 1,2,...,m).$ Note that the plastic strains satisfy condition (19).

In the case where the isotropy plane of a transtropic body is at a certain angle to the horizon, i.e. in case of $\varphi > 0$, the result of the system of equations for finding unknown coefficients A_{k0} and A_{kn}, according to conditions (25) has the form:

$$\left\{ \begin{array}{l} \mathrm{Re} \displaystyle\sum_{k=1}^{2} \mu_k^2 \left[A_{ko} + \sum_{n=1}^{N} A_{kn} \, \gamma_{kn,j}\left(z_{kj,n}\right) \right] = 0.5 \, \sigma_{xj}^p, \\[2ex] \mathrm{Re} \displaystyle\sum_{k=1}^{2} \left[A_{ko} + \sum_{n=1}^{N} A_{kn} \, \gamma_{kn,j}\left(z_{kj,n}\right) \right] = 0.5 \, \sigma_{yj}^p, \\[2ex] \mathrm{Re} \displaystyle\sum_{k=1}^{2} \mu_k \left[A_{ko} + \sum_{n=1}^{N} A_{kn} \, \gamma_{kn,j}\left(z_{kj,n}\right) \right] = 0.5 \, \tau_{xy_j}^p, \end{array} \right.$$

(30)

where

$$\gamma_{kn,j}\left(z_{kj,n}\right) = -(2n-1)_{\zeta_{kj}}^{-(2n-1)} \cdot \left(\sqrt{z_k^2 - a^2 - \mu_k^2 b^2}\right)^{-1},$$

$$\left(k = 1.2; \ n = 1.2,...,N; \ j = 1.2,...,m+2\right).$$

Right sides of equations (27) and (30) as statically determinable problems are represented by expressions (23) or (24).

To solve nonlinear systems (27) and (30), Gauss method is used. As shown, the method fits together if the initial (zero) approximation is taken close enough to the desired solution.

Let some initial approximation be known for points on beams $\theta_{\zeta j}$, i.e., be specified. $\rho_{\zeta j} = \rho_{\zeta j}^{(0)}$ $(j = 0,1,2...m+1)$. Inserting these values into $\rho_{\zeta j}^{(0)}$ using relations (28) and expression (23) or (24), depending on the plasticity condition in question, let us define components of plastic strains at selected points. Then, describing conditions (25) for $\rho_{\zeta j}^{(0)}$ in all $(m+2)$ directions with $\theta_{\zeta j} = const$ $(j = 0,1,2,...,m+2)$, we obtain a system of equations in relation to coefficients $\underline{A_{kn}}$ (k=1, 2; n=0, 1, 2, m+1). Assume that $A_{k0}^{(0)}$ will be the solution of this system.

Inserting found coefficients $A_{k\,II}^{(0)}$ into (29) or into the third equation of system (30) we obtain a "discrepancy" between the right and left sides; let us denote them as $\delta_j^{(0)} = \tau_{xy_j}^e - \tau_{xy_j}^p$ $(j = 1,2,...m)$. Consequently, the problem lies in selecting such values $\rho_{\zeta j}^{(k)}$, where values $\delta_j^{(k)}$ or some combination of $\delta_j^{(k)}$, for example $\Delta^{(k)} = \left(\sum_{j=1}^m \delta_j^{(k,2)}\right)1/2$, would be less than previously defined values ε_1 or ε_2, i.e., $\delta_j^{(k)} < \varepsilon_1$, or $\Delta^{(k)} < \varepsilon_2$; ε_1 or ε_2 are small values that characterize accuracy of the solution.

Depending on the sign $\delta_j^{(0)}$ for each direction $\theta_{\zeta j}$ values $\rho_{\zeta j}^{(1)} = \rho_{\zeta j}^{(0)} \pm \Delta\rho\varsigma$, change ($\Delta\rho\varsigma$ is taken with the plus sign, if $\Delta\delta_j^{(0)} > 0$, and the minus sign if $\Delta\delta_j^{(0)} < 0$, and again we determine the coefficients $A_{k\,II}^{(1)}$ beforehand by calculating right sides of the system (4.30) by

formulas (4.23) or (4.24). Then discrepancies $\delta_j^{(1)}$ (or $\Delta^{(1)}$) are calculated. If $\delta_j^{(1)} \le \varepsilon_1$ (or $\Delta^{(1)} \le \varepsilon_2$), the calculation ends where $\varepsilon_1 = 0,05; \ \varepsilon_2 = 0,01$.

But if $\delta_j^{(1)} > \varepsilon_1$ (or $\Delta^{(1)} > \varepsilon_2$), then the iterative process of calculation is repeated while changing the values $\rho_{\zeta j}^{(2)}$.

The calculation ends either when the required accuracy is reached, or when subsequent iterations do not lead to a reduction in values $\delta_j^{(k)}$ (or $\Delta^{(k)}$).

Numerical calculations

The calculations have been made in dimensionless variables

$$x|R_0, \ y|R_0, \ \sigma_x|\sigma_c, \ \sigma_y|\sigma_c, \ \tau_{xy}|\sigma_c, \ \sigma_x^{(\infty)}|\sigma_c, \ \sigma_y^{(\infty)}|\sigma_c, \ A_{k0}|\sigma_c, \ A_{kP/\sigma_c};$$
(31)

After reaching a predetermined accuracy strain at the infinity are determined from relations (16) with expressions:

(32) $\qquad \sigma_x^{(\infty)}, \sigma_y^{(\infty)}, \tau_{xy}^{(\infty)} \qquad z_k \to \infty$

$$\sigma_x^{(\infty)} = -\rho = -2\left(A_{10}\beta_1^2 + A_{20}\beta_2^2\right),$$
$$\sigma_y^{(\infty)} = -q = -2\left(A_{10} + A_{20}\right),$$
$$\tau_{xy}^{(\infty)} = 0 \ with \ \varphi = 0; \quad \tau_{xy}^{(\infty)} = -r = -2\,\text{Re}(A_{10}\beta_1 + A_{20}\beta_{22}) \ with \ \varphi \succ 0.$$

coordinates of elasticoplastic boundary-by formulas

$$x_j = \frac{a+b}{2}\left(\rho_{\zeta j} + \frac{a-b}{a+b}\right)\cos\theta_{\zeta j},$$

$$y_j = \frac{a+b}{2}\left(\rho_{\zeta j} + \frac{a-b}{a+b}\right)\sin\theta_{\zeta j},$$
(33)

where $\rho_{\zeta j}$ are the radii of the unit circle, $\theta_{\zeta j}$ is direction of beams.

Calculations show that quality pictures of configuration of elasticoplastic boundaries in both terms of instability coincide. However, for almost same areas of plasticity, laying depths (H) of mine openings are different. As an example, for yield conditions of Coulomb-Mohr from solutions of the system (30) with coefficients $A_{10} = 2,7174$ and $A_{20} = -9,4680$, the strains at infinity are: $-P_k = -p/k_0 = 9,620$ and $-Q_k = -q/k_0 = 13,50$. If we assume that $\gamma = 2,5\dfrac{T}{M^2}$ (average density of rock massif) and

$k_0 \approx \sigma_c = 200 \dfrac{Tc}{m^2}$, then from the equation $\gamma H_k = q / k_0 = 13,50$ we can determine the laying depth of the mine opening, for conditions of Coulomb-Mohr, i.e., $H_k = 1080м.$ .Similarly, from the solution to the system (30) with coefficients $A_{10} = 2,2134$ and $A_{20} = -8,0285$, strains at infinity $-P_x = -p/\sigma_c = 7,43$, $-Q_x = -q/\sigma_c = 11,63$. For comparison of laying depth in the conditions of Coulomb-Mohr yield and Hoek-Brown let us assume that $\gamma = const,$ then $\gamma H_x = q/\sigma_c = 11,63$, consequently the laying depth of the mine opening, for conditions of Hoek-Brown $H_x = 930,4m.$ The difference between the depths of $(H_k - H_x)$ is $\Delta H = 149.6m.$

If you seek to keep all other conditions equal, for both conditions of plasticity, i.e., keep the laying depth of the cavity at 1080 m, then for these yield conditions, dimensions of inelastic deformation area will be different.

This phenomenon is observed by means of an iterative process with adjustments $+\Delta a$ and $+\Delta в$ with achieving accuracy $\left|P_k - P_x\right| \le \varepsilon$, and $\left|Q_k - Q_x\right| \le \varepsilon$ and $\Delta \le \varepsilon$ ($\varepsilon = 0,01.$).

As can be seen from Figure 2 (Curve - 2), zone of plasticity under Hoek-Brown conditions with other equal ($H_k = 1080m = constant$) conditions, compared to the plasticity area in relation of Coulomb-Mohr yield, tends to increase.
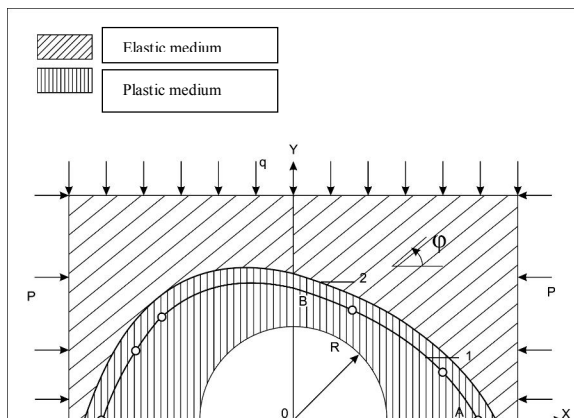

Fig.2. Comparison of inelastic deformation area size for various plasticity conditions

$\varphi = 30^0$; $\beta_1 = 2,0$; $\beta_2 = 0,8.$
$H_k = 1080m = constant.$

Curve 1 corresponds to the Coulomb-Mohr condition;

curve 2 corresponds to the Hoek-Brown condition.

Now let us consider the case where in expression (24) the value of internal pressure is not zero i.e., Pi ≠ 0 (Pi = P0). All other conditions being equal ($\gamma H = constant$). The iterative process is accompanied with adjustments $+\Delta a$ and $+\Delta b$.

Figure 3 shows configuration of the inelastic deformation area depending on various values of internal pressure.
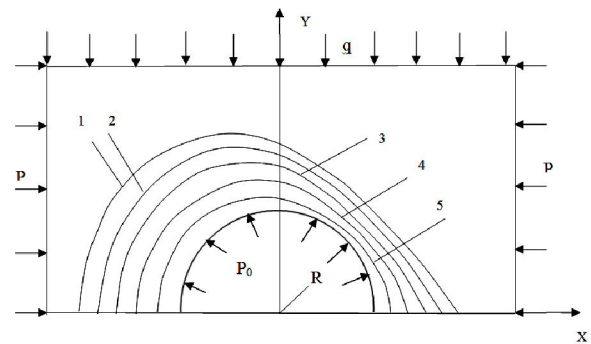

Figure 3. Configurations of the inelastic deformation area for isotropy plane

$\varphi = 45^0$ depending on various values of internal pressure. The parameters of elastic anisotropies $\beta_1 = 2,0, \beta_2 = 0,8$ and strains at the infinity $-\dfrac{p}{\sigma_c} = 7,431 = cons\tan t,$ $-\dfrac{q}{\sigma_c} 11,630 = cons\tan t.$

Curve 1 corresponds to

$\dfrac{p_0}{\sigma_c} = 0,$ $2 - \dfrac{p_o}{\sigma_c} = -0.1,$

$\dfrac{p_0}{\sigma_c} = 0.2,$ $3 - \dfrac{p_o}{\sigma_c} = -0.3,$

$4 - \dfrac{p_o}{\sigma_c} = -0.4.$

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References
1. Nottrot, R., 1967. On the numerical solution of the plane elastoplastic problem. J.Engng Math, Vol. 1, 2.
2. Nottrot, R. and R. Timman, 1967. General method of solving the plane elasto-plastic problem. -J.Engng Math, Vol. 1, 1:.
3. Southwell, R. and D.N. de Allen, 1950. Relaxation methods applied to engineering problems. IV. Philos, Trans. Roy. Soc. London, Ser. A, 242:
4. Perlin, P.P., 1960. Approximative method for solving of elasticoplastic problems. Engineering magazine, 28: 145-150.
5. Perlin, P.P., 1960. Elasticoplastic distribution of strains around the mine opening. In the Proceedings of MIPT, 5: 30-40.
6. Aytaliev, S.M., M.E. Eskaliev and J.K. Massanov, 1981. On elasticoplastic distribution of strains and displacements in an anisotropic massif with a mine opening. In the Applied problems of strength and plasticity. Gorky: the Editing House of the Gorky University, pp: 129-136.
7. Eskaliev, M.E. and J.K. Massanov, 1983. On elasticoplastic state of an anisotropic body with a mine opening. In the Mechanics of tectonic processes. Alma-Ata: Nauka, pp: 152-166.
8. Eskaliev, M.E., 1996. Effect of rock mass dilatancy on the elasticoplastic state of a mine opening in transtropic rock massif. In the Proceedings of the Academy of Sciences of the Kazakhstan Republic Physico-Mathematical Series, 3: 72-78.
9. Erzhanov, J.S, Sh.M. Aytaliev and J.K. Massanov, 1971. Stability of horizontal mine openings in an inclined layered massif. Alma-Ata: Nauka, pp: 160.
10. Lekhnitsky, S.G., 1977. Theory of an anisotropic body elasticity. Moscow, pp: 415.
11. Brown, E.T. and E. Hoek, 1980. Underground excavations in rock, Intuition of mining and metallurgy.

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