Modeling the real sector of the economy within the context of general economic equilibrium models

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Abstract. This article is about methods of modeling production sector for computable general equilibrium modeling. This article presents key different production functions, techniques of their modifications for including new sectors, different strategies of agents in production sector.
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Introduction

Constructing prognostic economic-mathematical models is among the most popular and sought-after problems in applied macroeconomics. Constructed models can be used in obtaining forecasts based on which development strategies and state programs are developed, budgets are planned out, etc. Another area economic-mathematical models are used in is scenario forecasting ("what will happen if"), which helps pre-compute the potential consequences of various managerial actions.

There are numerous mathematical methods for forecasting: time series analysis, regression analysis, the inter-industry balance model, the theory of general economic equilibrium, etc. Each method has its own area of application, limitations, strengths and weaknesses. Thus, for instance, it would be incorrect to use extrapolation methods and time-series analysis (the trend, the Box-Jenkins model) for long-term forecasting, since they do not adjust for structural changes taking place in the modeled environment over time. Thus, one of the major tasks faced by forecast developers is the choice of which forecast methodology to use. The wrong choice of methodology may result in below-par forecasts.

Main part

One of the optimum methods for obtaining integrated mid- and long-term forecasts of the state's social-economic development is the methodology of general economic equilibrium (hereinafter "GEE"). This approach's distinctive characteristics, which were identified on the strength of an analysis of works dedicated to this issue ([1], [2], and [3]), are:

1. The rational conduct of all the economic agents, which is described through the goal-setting problem, each agent aspires to resolve its local problem. From the mathematical standpoint, an optimization problem with limitations is formulated for each agent.
2. An integrated approach to the economic system modeled: searching for simultaneous equilibrium in all markets examined in the models – not for a set of private equilibriums in some markets considered in isolation from each other.
3. Economically substantiated non-linear forms of dependence.
4. A low sensitivity to the depth of retrospective data.

This work's aim is to establish an economic-mathematical arrangement describing the process of modeling the real sector in general economic equilibrium models, which can be used both by economists for developing models in the class of general economic equilibrium models and for developing specialized programming tools for working with GEE models. The [4] analysis of known programming products for working with GEE models revealed there is no product on the market that would concurrently possess a powerful computational kernel to describe and effectively compute GEE models and also possess "Business Intelligence" functionality, which includes integration with databases, an intuitively user-friendly interface, scenario analysis, report form construction, etc.

In constructing a model, it is important to find a reasonable balance between the model's complexity and its resemblance to the real world. One can try to describe everything in a model, even less significant associations and impacts, but a model like that is likely to be hard to manage and too complex to perform calculations. The economic modeling process based on the theory of general economic equilibrium is no exception. The most optimum approach to constructing GEE models is the gradual complication of models via the addition of new agents and associations. Thus, the process of constructing GEE models can be viewed as putting together some sort of a construction set consisting of a number of little bricks each of which is a specific sector of the economy and a market operating within the economy. This work addresses standard functions and problems used in GEE modeling, as well as possible modifications associated with the addition of new sectors, agents, and markets into models. The programming product implemented based on this economic-mathematical arrangement will make it possible to perform the visual construction of GEE models based on such bricks both for instructional purposes and for constructing real models.

The real sector is one of the economy's key sectors examined in absolutely all known GEE models. In this work, the authors provide methods and ways to model real sector agents in GEE models.

The activity of **real sector** agents is described by the production function. It should be noted that production functions are used both at the micro-level, for describing specific manufacturers' activity that is associated with the choice of an optimum set of factors [5] and is a consequence of implementing an optimum production program [6], and at the macro-level, for describing entire industries within the economy. This work focuses on the issue of using production functions at the macro-level.

Currently, there are quite a number of various types of production function modifications, the most popular of which are: the constant elasticity of substitution (CES) function, the Cobb-Douglas function, which is a particular functional form of the CES function, the linear production function, which is also a particular form of the CES function, and the Leontief fixed factor proportions function.

In practice, the choice of a specific functional form is made based on suppositions and the type of the economy as well as econometric studies. Table 1 provides the key characteristics of these functions, which facilitate the choice of a function.

We should note the areas of application of these functions. The Leontief function, for instance, can be applied to processes, for which there is a strictly defined production technology and which do not allow deviation from technological norms. Such norms can exist in small-scale production or team production but not in a market economy. Therefore, this function is not suitable for describing the activity of entire industries operating within the market economic system.

Another type is the linear function. It works well for modeling national economy industries, yet its substantial drawback is perfect factor substitutability.

Thus, when it comes to GEE modeling, the most suitable functions for modeling the real sector are the Cobb-Douglas function and the CES function.

The activity of manufacturers, according to production theory, can be described via the cost minimization problem (1) or through profit maximization (2) if there is a delimiter in the form of a production function.

Table 1. The key characteristics of production functions

	Factor substitution elasticity	**Homogeneity**
Cobb-Douglas function	Const (=1)	Homogeneous
CES function	Const	Homogeneous
Linear function	Infinity	Homogeneous at constant term = 0
Leontief function	0	Homogeneous

$$\begin{cases} TC -> \min \\ Q = F(\mathrm{Re}\,s_0,.......\mathrm{Re}\,s_i) \end{cases}, \quad (1)$$

where TC are the manufacturer's total costs,

Q is the production volume,

F ($\mathrm{Re}\,s_i$) is the volume of using the i[th] production resource.

$$\begin{cases} \pi -> maz \\ Q = F(\mathrm{Re}\,s_0,.......\mathrm{Re}\,s_i) \end{cases}, \quad (2)$$

where π is the manufacturer's profit.

$$\begin{aligned} &\sum_{i=1}^{n} P_{res_i} * \mathrm{Re}\,s_i^j \quad > \min \\ &Q^j = A^j * \prod_{i=1}^{m} \mathrm{Re}\,s_i^{j\,\alpha_i^j} \end{aligned}, \quad (3)$$

where Q^j is the volume of products turned out in the j[th] industry,

P_{res_i} is the price of the j[th] production resource,

$\mathrm{Re}\,s_i^j$ is the volume of using the i[th] production resource in the j[th] industry,

m is the number of production resources used,

A^j is the scaling coefficient of the Cobb-Douglas production function

α_i^j are the parameters of the Cobb-Douglas production function (they correspond to the elasticity of output in terms of the i[th] production resource in the j[th] industry).

Within the GEE context, this problem should be solved analytically. The result of solving this problem should come as functions of demand for production resources. Ultimately, in establishing the equation system to describe equilibrium in all markets examined in the model, the functions of demand obtained at this stage will be equalized with the supply of these resources. For the analytic solution of the problem (3) we used the Lagrange multiplier method. The result of solving this problem – the function of demand for the resource – is expressed via the following function (4).

$$Res_i^j = A^{j\ 1} * Q^j * \prod_{k=1}^{m} \frac{P_{res_k}}{\alpha_k^j}^{\alpha_k^j \frac{1}{\sum_{k=1}^{m}\alpha_k^j}} \frac{\alpha_i^j}{P_{res_i}},$$

The provided "standard" version of the Cobb-Douglas function does not let one adjust for the scientific-technical progress (STP) factor which helps describe processes of accelerated innovation development. The significance of such processes to modeling and forecasting, especially long-term forecasting, which GEE models are aimed at, is illustrated in [7]. There are several ways to make allowances for this factor in GEE models [8]. One of the ways is adding a multiplier into the production function.

$$Q^j = A^j * e^{\beta^j * t} * \prod_{i=1}^{m} \operatorname{Re} s_i^{j\,\alpha_i^j}, \qquad (5)$$

where β^j is a parameter indicating the level of STP in the j[th] industry,

t is the time index.

It is apparent that changes in the form of the production function will entail changes in the problem (3) as well as changes in the function of demand for resources, which was obtained as a result of solving the problem:

$$Res_i^j = \left[A^{j^{-1}} * Q^j * e^{-(\beta^j * t)} * \prod_{k=1}^{m} \left(\frac{P_{res_k}}{\alpha_k^j} \right)^{\alpha_k^j} \right]^{\frac{1}{\sum_{k=1}^{m} \alpha_k^j}}$$

, (6)

As was noted above, there is an alternative manufacturer strategy – profit maximization (the problem (2)).

In case of using the Cobb-Douglas function, the problem (2) will take on the following form:

$$\begin{cases} PX^j * Q^j - \sum_{i=1}^{m} P_{res_i} * \operatorname{Re} s_i^j -> \max \\ Q^j = A^j * \prod_{i=1}^{m} \operatorname{Re} s_i^{j\,\alpha_i^j} \end{cases},$$

where PX^j is the cost of a unit of the produced j[th] product, which is established by the manufacturer; the rest of the denotations are equivalent to those in the cost minimization problem.

As a result of solving the problem (7) using the Lagrange multiplier method, we get the function of demand for production resources:

$$Res_i^j = \frac{\alpha_i^j * Q^j * PX^j}{P_{res_i}}$$

In adding the STP factor into the model, the production resource demand function resulting from solving the profit maximization problem does not change and remains the same. (4)

We obtained in a similar way production resource demand functions in using the CES production function expressed via the following formula (9):

$$Q^j = A^j * \left(\sum_{i=1}^{m} \gamma_i^j \operatorname{Re} s_i^{j\,\frac{\delta^j - 1}{\delta^j}}\right)^{\frac{\delta^j}{\delta^j - 1}}, (9)$$

where A^j is the scaling coefficient in the j[th] industry,

γ_i^j is the weighting coefficient at the i[th] factor in the j[th] industry,

δ^j is the elasticity coefficient.

Table 2 provides production resource demand functions depending on the type of production function. Denotations used in the table have been explained above.

Table 2. The function of demand for resources depending on the chosen production function and manufacturer strategy

Profit maximization		
Cobb-Douglas function	without STP	$Res_i^j = \frac{\alpha_i^j * Q^j * PX^j}{P_{res_i}}$ (8)
	with STP	
CES	without STP	$Res_i^j = A^{j^{\delta^j - 1}} * (\gamma_i^j * PX^j / P_{Res_i})^{\delta^j}$ (10)
	with STP	$Res_i^j = (A^j * e^{\beta^j * t})^{\delta^j - 1} * (\frac{\gamma_i^j * PX^j}{P_{Res_i}})^{\delta^j}$ (11)
Cost minimization		
Cobb-Douglass function	without STP	$Res_i^j = \left[A^{j^{-1}} * Q^j * \prod_{k=1}^{m} \left(\frac{P_{res_k}}{\alpha_k^j}\right)^{\alpha_k^j} \right]^{\frac{1}{\sum_{k=1}^{m}\alpha_k^j}} \frac{\alpha_i^j}{P_{res_i}}$ (4)
	with STP	$Res_i^j = \left[A^{j^{-1}} * Q^j * e^{-(\beta^j * t)} * \prod_{k=1}^{m} \left(\frac{P_{res_k}}{\alpha_k^j}\right)^{\alpha_k^j} \right]^{\frac{1}{\sum_{k=1}^{m}\alpha_k^j}} \frac{\alpha_i^j}{P_{res_i}}$ (6)
CES	without STP	$Res_i^j = Q^j * A^{j^{-1}} \left[\sum_{k=1}^{m} \gamma_k^j \left(\frac{\gamma_k^j P_{\operatorname{Re} s_i}}{\gamma_i^j P_{\operatorname{Re} s_k}} \right)^{\delta^j - 1} \right]^{\frac{\delta^j}{1 - \delta^j}}$ (12)
	with STP	$Res_i^j = Q^j * A^{j^{-1}} * e^{-(\beta^j * t)} * \left[\sum_{k=1}^{m} \gamma_k^j \left(\frac{\gamma_k^j P_{Res_i}}{\gamma_i^j P_{Res_k}} \right)^{\delta^j - 1} \right]^{\frac{\delta^j}{1 - \delta^j}}$ (13)

(7)

The functions provided in Table 2 must be changed in including additional sectors. Thus, for instance, in adding an agent modeling the role of the **state** into the model, prices for production resources must be raised by the size of the tax rate. Instead of using the variable $P_{\operatorname{Re} s_i}$, in all the resulting functions of demand one has to use the expression:

$$P_{\mathrm{Re}\,s_i} * (1 + Tax_{\mathrm{Re}\,s_i}), \quad (14)$$

where $Tax_{\mathrm{Re}\,s_i}$ is the size of the tax rate for the i[th] production resource.

Another important aspect pertaining to the conduct of real sector agents is modeling international trade. Issues in modeling the real sector in adding the "outside world" agent into the model are examined in [9].

One more possible way to modify target problems describing manufacturer strategies, and, consequently, obtained functions of demand for resources, is associated with adjusting for intermediate consumption, which is based on inter-industry balance (IIB). In this case, the firm's costs are not only costs related to production resources but costs related to intermediate products. Here, instead of the product price PX^j, which is used in the functions (8), (10), (11), we need to use the expression

$$PX^j - \sum_{i=1}^{n} a_{ij} * PC^i, \quad (15)$$

where a_{ij} are the technological coefficients of inter-industry balance,

n is the number of manufacturers examined in the model,

PC^i is the cost of the i[th] product on the market.

Conclusion

The methodology for modeling the real sector provided in this article was used in developing the GEE model programming tools P5-CGEM, registered in the Federal Service for Intellectual Property, Patents, and Trademarks under No. 2009615853. Through the use of the P5-CGEM tools, a number of GEE models were developed: the model for the innovation development of the Republic of Kazakhstan in the interests of the Ministry of Economic Development and Trade of the Republic of Kazakhstan and the equilibrium model of the Russian Federation [10]. The programming product was implemented inclusive of the above approaches.

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References

1. Lofgren, H., R.L. Harris, and S. Robinson, 2002. A Standard Computable General Equilibrium Model in GAMS. International Food Policy Research Institute, pp: 69.
2. Freixas, X. and J.-C. Rochet, 1997. Microeconomics of Banking. Massachusetts Institute of Technology, pp: 312.
3. Bakhtizin, A., 2003. The "Russia: The Center – Federal Districts" Computable Model. Moscow: TsEMI RAN.
4. Perlov, M.S., D.N. Schultz and M.N. Schultz, 2012. Computable General Equilibrium Models: Programming Implementation and Numerical Methods. European Social Science Journal, 10(1): 428-436.
5. Dolgova, Ye.V and R.A. Fayzrakhmanov, 2005. Situation Identification in Managing the Material Flows of Enterprises. Pribory i Sistemy. Upravleniye, Kontrol, Diagnostika, 5: 60-62.
6. Kushnirsky, F.I., 2001. A Modification of the Production Function for Transition Economies Reflecting the Role of Institutional Factors. Comparative Economic Studies, 43(1): 1-30.
7. Perlov, M.S., R.A. Fayzrakhmanov, and Ye.V. Dolgova. Modeling National Economy Indicators under the Impact of the Innovation Development Factor. Electronic Scientific Journal "Upravleniye Ekonomicheskimi Sistemami", www.uecs.ru/predprinematelstvo/item/1379-2012-06-04-06-46-38.
8. Bayzakov, S.B., 2010. Alternatives to Economic Growth: The Innovation and Evolutional Development of the Russian Economy. In the Proceedings of the 3rd Scientific Readings for Economist-Professors and PhD Candidates (Yekaterinburg, February 2-4, 2010). Izdatelstvo Uralskogo Gosudarstvennogo Ekonomicheskogo Universiteta.
9. Shoven, J.B. and J. Whalley, 1984. Applied General-Equilibrium Models of Taxation and International Trade: An Introduction and Survey. Journal of Economic Literature, 22: 1007-1051.
10. Schultz, D.N. and M.N. Schulz, 2010. The General Economic Equilibrium Modeling of Russia's Economy. Informatsionnyye Sistemy i Matematicheskiye Metody v Ekonomike: Sbornik Nauchnykh Trudov. Persmkii Gosudarstvennyi Universitet, Perm, Issue 3, pp: 122-136.

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