Lagrangian Particle Tracking: Model Development

M. Mahdavimanesh1, A.R. Noghrehabadi2, M. Behbahaninejad 2, G. Ahmadi3, M. Dehghanian4
1Department of Mechanical Engineering, Neyriz Branch, Islamic Azad University, Neyriz, Iran
2Department of Mechanical Engineering, Shahid Chamran University of Ahvaz, Ahvaz, Iran
3Department of Mechanical and Aeronautical Engineering, University of Clarkson, New York, US
4Department of Mechanical Engineering, Marvdasht Branch, Islamic Azad University, Marvdasht, Iran

mmmahdavimanesh@gmail.com, a.r.noghrehabadi@scu.ac.ir, bnmorteza@scu.ac.ir, gahmadi@clarkson.edu, dehghanian@miau.ac.ir

Abstract: The study of micro and nano particle-laden multiphase flow has received much attention due to its occurrence in a wide range of industrial and natural phenomena. Many of these flows are multi-dimensional systems involving strong mass, momentum and energy transfer between carrying fluid and particulate phase. The purpose of the present paper is to survey brief description of Eulerian-Lagrangian modeling of two-phase flow.


http://www.lifesciencesite.com, 5

Keywords: Eulerian-Lagrangian framework; Turbulent two phase flow; Particle tracking; Two-way coupling

1. Introduction

A two-phase flow is defined as combination of continuous phase, e.g. gas or liquid, and disperse phase, e.g. particles or droplets. They are found in many industrial applications such as cyclone separators, jet mills, deposition in duct or pipe flows (most probably undesired deposition), dust precipitation or coating processes (desired deposition). They also occur in nature, e.g., desert sand storms, pollen in air and dispersion of pollutants. In order to model such a phenomenon, two theoretical approaches are considered, namely the Eulerian-Eulerian (two-fluids) and the Eulerian-Lagrangian approaches. The Eulerian description assumes continuum medium for discrete phase and solve conservative laws for both solid phase and gas phase [1-3]. With the rapid development of computational capabilities, the mixed Eulerian–Lagrangian approach attracted more and more attentions from many researchers. In this approach, the detailed particle motion behavior, which facilitates a better understanding of the physical phenomena, can be revealed by solving the Newtonian motion Equations in Lagrangian coordinate while solving continuity and momentum equations for continuum phase.

In this review we will focus on turbulent Lagrangian description of particle tracking to model two-phase flow and expose development process in modeling such phenomena.

2. Characteristics of Lagrangian description in turbulent flow

Empirical probes have shown that three impacts have to be allowed for, to predict particle dispersion precisely.

2.1 The inertia effect (IE)


2.2 The crossing trajectory effect (CTE)

This effect Udine [7], whereby particle dispersion is reduced in the presence of strong body forces due to particles rapidly passing through eddies.

2.3 The continuity effect (CE)

The continuity effect Csanady [1] tells that the dispersion in the direction of the drift velocity exceeds the dispersion in the other two directions. The effect results from the fact that “longitudinal” and “transverse” length scales are different, which is in turn due to continuity equation of fluid motion [8].

3. Particle equation of motion

The surrounding fluid will interact with particles. The Lagrangian approach for the simulation of the disperse phase is based on
Newton’s equation of motion. To date a number of such forces have been implemented in discrete particle simulations. The reader can refer to [9, 10] for the historical development of such forces. The equation of motion of particles is given by

\[ m_p \frac{d \vec{u}_p}{dt} = \sum F \]  

Here, \( F \) is sum of all forces that acting on particles which is represented in Table 1.

<table>
<thead>
<tr>
<th>Forces</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drag</td>
<td>( \vec{F}_{\text{Drag}} = \frac{\pi d^2}{8} \rho \frac{c_D}{\rho_p} \left( \vec{V} - \vec{u}_p \right) \left( \vec{u} - \vec{u}_p \right) )</td>
</tr>
<tr>
<td>Gravity</td>
<td>( \vec{F}_{\text{gravity}} = m_p \vec{g} )</td>
</tr>
<tr>
<td>Saffman</td>
<td>( \vec{F}_{\text{Saffman}} = 1.61d^2 \rho_p \left( \mu_p \right) \frac{1}{2} \left[ \frac{1}{2} \left( \vec{u} - \vec{u}_p \right) \times \vec{a} \right] )</td>
</tr>
<tr>
<td>Magnus</td>
<td>( \vec{F}_{\text{Magnus}} = \frac{\pi d^2}{8} \rho_p \left[ \frac{1}{2} \left( \vec{V} - \vec{u}_p \right) \times \left( \vec{u} - \vec{u}_p \right) \right] )</td>
</tr>
<tr>
<td>Virtual mass force</td>
<td>( \vec{F}<em>{\text{Virtual mass force}} = C</em>{Vm} \frac{d}{dp} \left( \vec{u} - \vec{u}_p \right) )</td>
</tr>
<tr>
<td>Basset history</td>
<td>( \vec{F}_{\text{Basset history}} = \frac{3}{2} d \rho \sqrt{\frac{d}{dp}} \left[ \frac{d}{dt} \left( \vec{u} - \vec{u}_p \right) + \vec{a} \right] )</td>
</tr>
<tr>
<td>Brownian</td>
<td>( \vec{F}_{\text{Brownian}} = m_p \frac{\pi d^2}{8} \rho_p \left( \vec{V} - \vec{u}_p \right) )</td>
</tr>
<tr>
<td>Turbophoresis</td>
<td>( \vec{F}_{\text{Turbophoresis}} = -f \frac{\pi d^2}{8} \rho_p \left( \vec{V} - \vec{u}_p \right) )</td>
</tr>
<tr>
<td>Thermo-</td>
<td>( \vec{F}_{\text{Thermo}} = -f \frac{\pi d^2}{8} \rho_p \left( \vec{V} - \vec{u}_p \right) )</td>
</tr>
</tbody>
</table>

Table 1. Acting forces on particles

The difference between the velocity of carrying fluid and of a particle moving in the carrying fluid causes the Drag force. The effect of gravity force on particle motion should be included, in the case where the free-fall velocity of particles and the velocity of carrying fluid are the same order of magnitude. The non-uniformity of the profile of averaged velocity of carrier fluid results in Saffman lift force [11]. The Magnus force is due to the particle rotation. During particle motion in a fluid, particles of complex shape (a spherical) always rotate. The spherical particles will also rotate in a flow with a no uniform velocity profile. The added mass or virtual mass force is the inertia added to a particle because an accelerating or decelerating body must move some volume of surrounding fluid as it moves through it. The Basset history term is the drag caused by unsteady motion of the particle in a viscous medium. If the size of particle suspended in a fluid is very small, the motion of the particle affected by discrete nature of molecular motion of the fluid which is called the Brownian force. Turbophoresis force arises because of the non-uniformity of the profile of fluctuation velocity of carrier fluid. Thermophoresis force arises as a result of the non-uniformity of the temperature profile of carrier fluid.

4. Fluid-phase flow model

The conservation equations for the fluid flow are given by (in tensorial notation):

\( (\rho_p \phi)_{,j} + (\rho_p U \phi)_{,j} = (\Gamma \phi)_{,j} + S_{\phi} + S_{\phi'} \)

Where, \( \rho_p \) is the gas density, \( U \) are the Reynolds-averaged velocity components, and \( \Gamma \) is an effective transport coefficient. The source terms \( S_{\phi} \) and \( S_{\phi'} \) are arisen from the transport equation and presence of particles, respectively. The \( k \) (energy contained in velocity fluctuations)–\( \varepsilon \) (rate of transfer of kinetic fluctuation energy to heat by viscous friction) model is widely used for the simulation of turbulent fluid flows in practical applications. Error! Reference source not found. and Table 2. Summary of terms in the general equation for the different variables that describe the fluid phase in Cartesian flow demonstrate the different variables and source terms that describe the fluid phase; respectively. The direct influence of the dispersed phase on the continuous phase is usually taken into account by

http://www.lifesciencesite.com 35  lifesciencej@gmail.com
formulating appropriate source terms for all quantities under consideration. In the situation which mass loading of particles is low, the influence of particles on their carrier phase is negligible. Thus, the source terms that appear in the conservative equations as a result of particles are zero. In many situations, the mass loading particle to fluid ratio is too large to allow one to be satisfied with the one-way approach discussed in the previous subsection.

\[
\begin{array}{ccc}
\phi & \Gamma & S_{\phi} \\
\hline
\text{Continuity} & 1 & 0 \\
\text{X-Momentum} & U & \mu + \mu_t \frac{\partial}{\partial x}(U) + \frac{\partial}{\partial y}(V) + \frac{\partial}{\partial z}(W) - \frac{\partial p}{\partial x} \\
\text{Y-Momentum} & V & \mu + \mu_t \frac{\partial}{\partial y}(U) + \frac{\partial}{\partial z}(V) + \frac{\partial}{\partial x}(W) - \frac{\partial p}{\partial y} \\
\text{Z-Momentum} & W & \mu + \mu_t \frac{\partial}{\partial z}(U) + \frac{\partial}{\partial x}(V) + \frac{\partial}{\partial y}(W) - \frac{\partial p}{\partial z} \\
\text{Turbulent Kinetic Energy} & k & \mu_t \frac{\partial}{\partial x} \left[ \frac{\partial}{\partial x}(U) + \frac{\partial}{\partial y}(V) + \frac{\partial}{\partial z}(W) \right] \\
\text{Viscous Dissipation Rate} & \varepsilon & \frac{\mu}{\sigma_\varepsilon} \left( C_k \varepsilon - C_\varepsilon \rho C \varepsilon \right) \\
\text{Production of Turbulent Kinetic Energy} & G_k & \mu_t \left[ \frac{\partial}{\partial x} \left( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \right) \right] \\
\text{Eddy Viscosity} & \mu_t & \rho C_k \varepsilon \frac{k^2}{\sigma_k} \\
\end{array}
\]

Table 2. Summary of terms in the general equation for the different variables that describe the fluid phase in Cartesian flow

\[
\begin{array}{ccc}
\phi & S_{\phi} & \text{Reference} \\
\hline
\text{Continuity} & 1 & 0 & \left[9, 20-22, 24, 25\right] \\
\text{X-Momentum} & U & \sum_{k=1}^{N_p} \sum_{n=1}^{N_T} \sigma_k \Delta L \left[ \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \right] \\
\text{Y-Momentum} & V & \sum_{k=1}^{N_p} \sum_{n=1}^{N_T} \sigma_k \Delta L \left[ \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} + \frac{\partial W}{\partial z} \right] \\
\text{Z-Momentum} & W & \sum_{k=1}^{N_p} \sum_{n=1}^{N_T} \sigma_k \Delta L \left[ \frac{\partial U}{\partial z} + \frac{\partial V}{\partial x} + \frac{\partial W}{\partial y} \right] \\
\text{Turbulent Kinetic Energy} & k & \sum_{k=1}^{N_p} \sum_{n=1}^{N_T} \sigma_k \Delta L \left[ \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \right] \\
\text{Viscous Dissipation Rate} & \varepsilon & C_{\mu} \frac{\rho C_k}{\sigma_k} \\
\text{Constants} & C_1 & 1.44; \ C_2 & 1.44; \ C_3 & 1.87; \ C_4 & 0.09; \ \sigma_k & 1.0; \ \sigma_\varepsilon & 1.3 \\
\end{array}
\]

Table 3. Fluid phase source terms in Cartesian flow

5. Lagrangian modeling

A common feature of most Lagrangian methods used to date is the decomposition of the driving fluid velocity into the mean and the fluctuating parts. The mean fluid velocity field results from the continuous phase computation and is interpolated at particle locations. Then, a Lagrangian model gives the fluctuating component. In the Euler-Lagrange approach the interaction between both phases requires an iterative solution procedure, which is usually called two-way coupling. The discrete form of Reynolds Stress Model equations and their relevant source terms were presented by Gouesbet et al[9] and Lain et al[10]. In addition to two-way coupling simulation, four-way coupling simulation accounts for inter-particle collisions which was considered by many researchers [10, 19-23].
instantaneous motion of particles with regard to their fluid neighborhood, and second, mean particle drift due to gravity.

5.1 Eddy Interaction Model
Hutchinson et al [26] considered particles which have one-directional radial motion in a turbulent pipe flow. Their well-known model is called eddy life time model. The model uses a stochastic approach to predict the characteristic of discrete phase. The adequate characteristic of the eddy interaction model lies in its simplicity and the fact that the only statistics required by the model are representative length, time and velocity scales, whereas, in the autocorrelation models (will discuss in the next section), the forms of either the temporal (Lagrangian) or the spatial (Eulerian) velocity auto-correlation functions (or both of these) are required. In single eddy interaction model (SEIM) each of a number of individual particles is tracked through a series of interactions with fluid eddies whose length, "lifetime" and velocity can all be random variables. Their model modified by some researchers to account for various particles dispersion [27-29].

The effective eddy interaction time interval \( t_e \) in which the velocities are kept constants are set to the minimum of the integral eddy life time \( T_e \) and the time scale for the crossing of the eddies \( t_c \) in order to account for the turbulence structure of the carrier phase as well as for the crossing trajectory effect[29] which is given by

\[
t_e = -t_c \ln(1 - \frac{L_c}{\tau_c [u - u_p]})
\]

(3)

In this approach[30, 31] as shown in Figure, at the start of the interaction time between fluid and particle \( (t = 0) \), the particle will be assumed to be sitting at the center of the eddy with velocity \( U_p \). During the interaction of eddy and non-fluid particle, instantaneous fluid velocity of eddy is remained constant in space and time within the eddy and is given by

\[
u = U + u^`
\]

(4)

Where \( u \) is the instantaneous eddy velocity, \( U \) is the mean velocity and \( u^` \) is the fluctuating velocity which is given by

\[
u^` = \sqrt{\frac{2k}{3}}
\]

Where \( \gamma \) is the zero mean unit variance Gaussian random vector.

With proceed of time both the eddy and the non-fluid particle have moved in space. The eddy moves with its instantaneous fluid velocity while the non-fluid particle movement is governed by the Newtonian equation of motion (Eq. (1)). The non-fluid particle remains under the influence of that eddy until the interaction time exceeds the eddy lifetime \( (T_e) \), or distance between the center of eddy and non-fluid particle \( (d(t_c)) \) exceeds the eddy length \( L_e \). So in this approach interaction time is given by

\[
t_e = \min(T_e, t_c)
\]

(6)

![Figure 1. Eddy interaction model (from[30, 31])](image)

Inertia and crossing trajectory and continuity effects have not been considered in original EIM. Some modifications have been taken to account by Graham[30, 31] to amend these deficiencies. The drawbacks of the method are twofold: a resulting velocity correlation coefficient is linear (rather than exponential) and the velocity record for a fluid particle is discontinuous (rather than continuous).

5.1.1 Eddy scale determination

As discussed earlier, to predict Particle motions in eddy interaction models three parameters are therefore determined: (i) eddy velocity, (ii) eddy lifetime and (iii) eddy length. Wang [32] have shown that the choice of eddy lifetime distribution in the eddy interaction model determines the form of the Lagrangian fluid velocity auto-correlation function.

Eddy scales used by Hutchinson and James et al [33] were obtained from empirical correlations using Laufer [34] pipe flow data. Gosman & Ioannides [27] assumed that the eddy length and lifetime are equal to the dissipation scales, given by

\[
T_e = \sqrt{\frac{3}{2}} \frac{C_{\mu} \frac{k}{\varepsilon}}{k}, \quad L_e = \sqrt{\frac{3}{2}} \frac{k_{\mu}}{\varepsilon}
\]

(7)

Where \( k \) and \( \varepsilon \) is the turbulence kinetic energy and its rate of dissipation, respectively and \( C_{\mu} = 0.09 \).

Eddy scales in the near-wall turbulence studied by Kallio&Reeks[35] where determined from laws of similarity at the wall. Milojevic[29] proposed the
following eddy length and time scales which is given by
\[ T_x = C_T \frac{k}{\varepsilon}, \quad L_x = \frac{\sqrt{2} C_T \frac{k^{3/2}}{\varepsilon}}{3} \]  
(8)

Typically a value of \( C_T \) is in the range of 0.2 to 0.96.

Call & Kennedy[36] accounted for anisotropic turbulence by the use of a Reynolds stress turbulence model for the primary flow, thereby allowing for different eddy velocities in different coordinate directions. Graham & James[31] investigated the performance of different eddy interaction models with random length and time scales.

### 5.2 Auto-correlation method

In analyzing turbulent two phase flow we almost deal with Lagrangian fluid velocity auto-correlation and Eulerian fluid velocity auto-correlation functions. These two functions represent correlations between fluctuations of velocity of carrier phase and have the following general form, respectively:

\[ R_L(t, \tau) = \langle u_i(t) u_j(t + \tau) \rangle \]  
(9)

\[ R_E(x, \tau) = \langle u_i(x, t_0) u_j(x + x, t_0 + \tau) \rangle \]  
(10)

Where \( u_i(t) \) and \( u_j(t + \tau) \) are the fluid velocity at time \( t \) and \( t + \tau \), respectively and angled brackets indicate ensemble averaging over many such particles. Lagrangian velocity coloration function is one of the most important statistics of turbulent flows. \( R_L \) should satisfy some requirements as following requirements:

I. \( \tau \to 0 \Rightarrow R_L \to 1, \quad \tau \to \infty \Rightarrow R_L \to 0 \)

II. \( (dR_L / d\tau)_{\tau=0} = 0, \quad \left( d^2 R_L / d\tau^2 \right)_{\tau=0} < 0 \)

Desjounguierz et al [37] proposed a Lagrangian method in which a correlation matrix is used in random process, which simulates the Lagrangian time correlations along the particle path. In the one dimensional formulation we have the following procedure in order to generate fluctuation velocities ([9, 38 and 39]):

Consider \( \mathbf{U} \) as the fluctuating velocity in the different time steps as follow

\[ \mathbf{U} = [ u_i', (0), u_i', (\Delta \tau), ..., u_i', (i \Delta \tau), ..., u_i', (n \Delta \tau) ] \]  
(11)

Define uncorrelated zero mean and unit variance Gaussian vector \( \mathbf{U} \) and \( \mathbf{Y} \) have the following relationship

\[ \mathbf{U}' = \mathbf{B} \mathbf{Y} \]

There is a symmetric, positive-definite matrix \( \mathbf{A} \) which has the following relationship with \( \mathbf{B} \) as follow:

\[ \mathbf{A} = \mathbf{B}^T \mathbf{B} \]

(13)

Where, \( \mathbf{B}^T \) is the transpose of \( \mathbf{B} \).

Element of matrix \( \mathbf{A} \) obtain simply by Frenkkel [40] family of the correlation as follow

\[ a_{ij} = \exp\left[ -\frac{|i-j|\Delta \tau}{(m^2 + 1)\tau_L} \right] \cos\left( \frac{m(i-j)\Delta \tau}{(m^2 + 1)\tau_L} \right) \]  
(14)

where \( m \) is the loop parameter and \( m = 1 \) is a recommended value[9]. In the final step we use Cholesky factorization in order to \( \mathbf{B} \) obtain from \( \mathbf{A} \)

### 5.3 Lagrangian temporal construction Model

Lu et al [41] proposed a Lagrangian model which represents satisfactory results in compared to experimental results. In their model, at time \( t \), the particle and a corresponding fluid point start out from the same position \( X_0 \) (Figure 2). After one time step, they arrive at \( X_i \) and \( X_p \), respectively, and the distance between them is \( \Delta s \). The relative coordinate system \( O-\Xi \Omega \Theta \) is chosen such that its original point, \( O \), is located at the position \( X_0 \), and the \( \Xi \) axis passes through the position \( X_p \) ([41-43]).

![Figure 2. The locations of the particle and a fluid particle (from[42])](http://www.lifesciencesite.com)

With the aid of time series analysis, the normalized fluctuating component in the location of particle in the \( i \)-direction of relative coordinate system \( O-\Xi \Omega \Theta \) is obtained by (no summation convention is used):

\[ W_i(X_{\rho}) = a_{i} b_i W_i(X_s) + \Psi_i \quad (i = 1, 2, 3) \]

(15)

Where

\[ W_i = \frac{u_i'}{\sqrt{a_i b_i}} \quad (i = 1, 2, 3) \]  
(16)

In Eq. (15), \( \Psi_i \) are mean zero, \( \sqrt{1-(a_i b_i)} \) variance Gaussian random numbers and \( a_i, b_i \) are
Lagrangian auto-correlation and Eulerian spatial velocity correlations functions, respectively which they for example pick from the Frenkel [40] family of auto-correlations as follow

\[ a_i = \exp\left(-\frac{\Delta t}{\tau_i^+}\right), \quad b_i = \exp\left(-\frac{\Delta s}{2\Lambda_i}\right) \cos\left(\frac{\Delta s}{2\Lambda_i}\right), \quad (i = l, 17) \]

Where are \( \Lambda \) the length scales and \( \tau_i^+ \) are the Lagrangian time scales.

5.4 Kraichnan Fourier modes

Kraichnan [44] suggested a simple method for generating a random field which resembles a pseudo-isotropic turbulence. In this approach fluctuation velocities are obtained as below

\[ \vec{u}(X_\rho, t) = \sqrt{\frac{2}{N}} \sum_{n=1}^{N} \vec{u}_i(\vec{k}_n) \cdot \cos\left(\vec{k}_n \cdot X_\rho + \omega_n t\right) \]  

Where

\[ \vec{u}_i(\vec{k}_n) = \vec{\zeta}_n \times \vec{k}_n, \quad \vec{u}_2(\vec{k}_n) = \vec{\zeta}_n \times \vec{k}_n \]  

(19)

And

\[ \vec{k}_n \cdot \vec{u}_1(\vec{k}_n) = \vec{k}_n \cdot \vec{u}_2(\vec{k}_n) = 0 \]  

(20)

Satisfy the incompressibility condition. The components of vectors \( \vec{\zeta}_n \), \( \vec{\zeta}_n \) and the values of frequency \( \omega_n \), were picked independently from a three or two dimensional Gaussian distribution with a standard deviation of unity. Each component of \( \vec{k}_n \), is a Gaussian random number with a standard deviation that depends on energy spectrum [44]. Here, \( N \) is the number of terms in the series that usually is considered as 100.

Some modifications were taken to account for anisotropy effects [21, 22 and 45].

5.5 Models based on the Langevin equation

A following stochastic differential equation has been proposed to model the behavior of fluid velocities

\[ \frac{du}{dt} = -u - \frac{u}{T_x} + \left(\frac{2\gamma^2}{T_x}\right)^{1/2} \psi(t) \]  

(21)

Here, \( \psi(t) \) is the Wiener process (white noise); it is a stochastic process of zero mean, \( \langle \psi(t) \rangle = 0 \), a variance equal to the time interval \( \langle (\psi(t))^2 \rangle = dt \). The above is the Langevin equation first proposed to model the Brownian motion; in that context, it represents the equation of motion of a small particle in surrounding fluid [46, 47]. Some models presented by Pozorski & Minier [46] which account for main three effects in Lagrangian simulation, i.e. IE, CTE and CE.

References


http://www.lifesciencesite.com

http://www.lifesciencesite.com

lifesciencej@gmail.com


4/2/2013