New approach to control of ball and beam system and optimization with genetic algorithm

Hamed Ebrahimi Mollabashi[1], Aref Shahmansoorian [2], Soroush Rastegarpour[3], Amir Hooshang Mazinan[4]

[1.] MSc student in Department of Electrical Engineering, Islamic Azad University (IAU), South Tehran Branch, Tehran, Iran. hamed_ebrahimi@aol.com
[2.] Assistant Professor, Department of Electrical Engineering, Imam Khomeini International University, Qazvin, Iran. shahmansoorian@ikiu.ac.ir
[3.] MSc student in Department of Electrical Engineering, Islamic Azad University (IAU), South Tehran Branch, Tehran, Iran. s.rastegarpour@yahoo.com
[4.] Assistant Professor, Department of Electrical Engineering, Islamic Azad University (IAU), South Tehran Branch, Tehran, Iran. mazinan@azad.ac.ir

Abstract: A kind of robust control strategy for a class of under actuated systems with mismatched uncertainties is proposed in this paper. In this approach, multiple layers sliding surface (MLSS), are defined, firstly, divided system states into several subsystems and the sliding mode surface of every subsystem is defined. Secondly, the sliding mode surface of one subsystem is selected as the first layer sliding mode surface. The first layer sliding mode surface is used to construct the second layer sliding mode surface with the sliding mode surface of another subsystem. This process continues till sliding mode surfaces of all the subsystems are included. Two methods are used for optimization of response, genetic algorithm (GA) and design compensator at the last layer of sliding mode surface. GA improved the coefficient of sliding mode surface, and compensator, compensated delaying with the mismatched uncertainties. In this paper a new sliding mode control law is designed to guarantee that every sliding surface can converge rapidly to zero. The asymptotic stability of the entire sliding mode surfaces is proved theoretically. The simulation results for the ball and beam system are presented to demonstrate the effectiveness and robustness of the method. This control scheme is compared with Decoupled sliding mode with fuzzy neural network control (DSMFNNC) scheme. The results show that multiple layers sliding mode control performance has better than (DSMFNNC).

[Hamed Ebrahimi Mollabashi, Aref Shahmansoorian, Soroush Rastegarpour, Amir Hooshang Mazinan. **New approach to control of ball and beam system and optimization with genetic algorithm**. *Life Sci J* 2013;10(5s):415-421] (ISSN:1097-8135). http://www.lifesciencesite.com. 75

Keywords: under actuated systems, multiple layers sliding surface, robustness, stability, genetic algorithm.

1. Introduction

Study of under actuated systems has rapidly expanded in recent years. Under actuated systems are characterized by the fact that they have fewer actuators than the degrees of freedom to be controlled. These systems with fewer actuators and gaining fault tolerance make for decreasing the number of actuators, lightening the systems, reducing the cost. There has been growing attention and increasing interest in under actuated systems in recent years. Mechanical systems with fewer number of control inputs than the number of degrees of freedoms to be controlled are called under actuated systems [1]. Class of typical under actuated systems, Ball and Beam systems are often used as a benchmark for verifying the effectiveness of new control approaches.

Ball and beam system is one of the most enduringly popular and important laboratory models for teaching control systems engineering [2]. It is widely used because many important classical and modern design methods can be studied based on it (is shown in Fig.2).

The sliding mode controller is a powerful nonlinear controller, which has been developed and applied to feedback control systems for the last three decades. For under actuated systems, designing a conventional sliding mode surface is not appropriate, because the parameters of the sliding mode surface can't be obtained directly according to the Hurwitz condition. Yi [3] presented a hierarchical sliding mode controller for large scale under actuated systems, whose sliding mode surfaces were asymptotically stable. Thus we can consider designing the multiple layers sliding mode control for ball and beam system. In multiple layers sliding surface, Firstly, divided system states into several subsystems and the sliding mode surface of every subsystem is defined. Secondly, the sliding mode surface of one subsystem is selected as the first layer sliding mode surface. The first layer sliding mode surface is used to construct the second layer sliding mode surface with the sliding mode surface of another subsystem. This process continues till sliding mode surfaces of all the subsystems are included. Two methods are used for optimization of response, genetic algorithm and design compensator at the last layer of sliding mode surface. Genetic algorithm improved the coefficient of sliding mode surface [4], and compensator, compensated delaying with the mismatched uncertainties.

2. Multiple layers sliding mode control design

For SIMO under actuated mechanical systems, the mathematical model can be translated into the following form

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_1 + b_1 u + d_1 \\
\dot{x}_3 &= x_3 \\
\dot{x}_4 &= f_2 + b_2 + d_2 \\
&\dots \\
&\dots \\
\dot{x}_{2n-1} &= x_{2n} \\
\dot{x}_{2n} &= f_n + b_n u + d_n
\end{aligned} \tag{1}$$

Where $X = [x_1, x_2, ...x_{2n}]^T$ is the state variables and u is the input of the system, f_n and b_n are bounded nominal d_n is the term of lumped mismatched uncertainties, including system uncertainties and external disturbances. It's assumed that they are bounded by$|d_n| \le d_{nmax}$, where d_{nmax} are upper boundary, i=1, 2, …,n.

As the Multiple layers sliding structure has been shown in Fig. 1

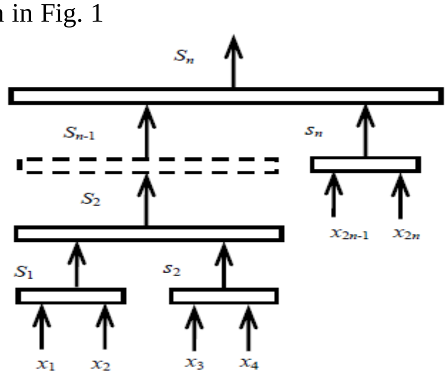


Fig 1. Multiple layers sliding structure

First layer can be defined by (1) where c_i are constants which have the same sign but this parameters and other sliding surface can't be obtain according Hurwitz condition so for this reason and optimizations, using genetic algorithm.

$$s_1 = c_1 x_1 + c_2 x_2 \tag{2}$$

The equivalent law control obtain from $\dot{s}_1 = 0$

$$0 = c_1 \dot{x}_1 + c_2 \dot{x}_2 \rightarrow c_1 x_2 + c_2 (f_1 + b_1 u) \tag{3}$$

$$u_{eq(1)} = -\frac{c_2 f_1 + c_1 x_2}{c_2 b_1} \tag{4}$$

According to the fig.1 the second layer defined as $s_2 = c_3 x_3 + s_1$ for the state variables of the ith subsystem, the sliding mode surface is defined as

$$s_i = c_{i+1} x_{i+1} + s_{i-1} \tag{5}$$

The total equivalent law control obtain from $\dot{s}_i = 0$,

so should be obtain $\dot{s}_i$

$$\dot{s}_i = \sum_{j=1}^{m} c_{2j-1} . x_{2j} + \sum_{j=1}^{m} c_{2j} (f_j + b_j u + d_j)$$
$$m = \begin{cases} (i+1)/2 & i, is \quad odd \\ i/2 & i, is \quad even \end{cases} \tag{6}$$

So the total equivalent law for ith layer sliding surface can be defined as:

$$u_{eq(i)} = -\frac{\sum_{j=1}^{m} c_{2j-1} . x_{2j} + \sum_{j=1}^{m} c_{2j} f_j}{\sum_{j=1}^{m} c_{2j} b_j} \tag{7}$$

u_{sm} can be assumed as:

$$U_{sm(i)} = u_{eq(i)} + u_{sw(i)} + u'_{sw(i)} \tag{8}$$

Because total equivalent law is not enough to guarantee that every sliding surface can converge rapidly to zero, $u_{sw(i)}$ is the switch control law for every layer sliding surface and $u'_{sw(i)}$ is the switch control law for the last layer sliding surface. The switch control law, $u_{sw(i)}$ and $u'_{sw(i)}$ can improved the response time.

The switch control law is defined as [5]:

$$u_{sw(i)} = \begin{cases} 0 & i = 1 \\ \sum_{j=1}^{i} \eta_j \operatorname{sgn}(s_j) / dem(i) & i > 1 \end{cases} \tag{9}$$

Where η is a positive constant, $\eta_j = 2\eta_{j-1}$,

$$den(i) = c_2 b_1 + \sum_{j=2}^{m} (c_{2j} b_j .\mathrm{sgn}(s_{2j-1})) \quad (10)$$

$$u'_{sw} = -k.s_{2n-1} / \sum_{i=1}^{n} c_{2n} b_n \quad (11)$$

For the matched uncertainties, the above multiple layer sliding mode control can resist them because of the invariant characteristic of the sliding mode. For the mismatched uncertainties, will be designed a sliding mode compensator to resist them where $u_{com(i)}$ is the distributed compensator, For the i-th layer sliding surface, $u_{com(i)}$ is given by[5]:

$$u_{com(i)} = \frac{\sum_{j=1}^{m} c_{2j} .\bar{d}_j \,\mathrm{sgn}(s_{2j-1})}{\sum_{j=1}^{m} c_{2j} b_j} \quad (12)$$

The total controller for the under actuated system (1) can define as:

$$u_{(i)} = u_{sm(i)} + u_{com(i)}$$

3. Stability

For the under actuated system (1), the Lyapunov function can be defined as [7]:

$$v_{2n-1} = \frac{1}{2} s^2_{2n-1} \rightarrow \dot{v}_{2n-1} = s_{2n-1}.\dot{s}_{2n-1}$$

$$\int_0^t \dot{v}_{2n-1} d\tau = \int_0^t (-k\, s^2_{2n-1} - s_{2n-1}.\sum_{i=1}^{2n-1} \eta_i .\mathrm{sgn}(s_i) - \sum_{i=1}^{n} (c_{2i} \bar{d}_i - c_{2i} d_i) | \mathrm{sgn}(s_{2i-1}) | d\tau \quad (13)$$

Both sides of (13) are integrated:

$$\int_0^t \dot{v}_{2n-1} d\tau = \int_0^t (-k\, s^2_{2n-1} - s_{2n-1}. \sum_{i=1}^{2n-1} \eta_i .\mathrm{sgn}(s_i) - \sum_{i=1}^{n} (c_{2i} \bar{d}_i - c_{2i} d_i) | \mathrm{sgn}(s_{2i-1}) | d\tau \quad (14)$$

$$v_{2n-1}(t) = v_{2n-1}(0) - \int_0^t (k\, s^2_{2n-1} + s_{2n-1}. \sum_{i=1}^{2n-1} \eta_i .\mathrm{sgn}(s_i) + \sum_{i=1}^{n} (c_{2i} \bar{d}_i - c_{2i} d_i) | \mathrm{sgn}(s_{2i-1}) |) d\tau < \infty \quad (15)$$

We know that: $\eta_j = 2\eta_{j-1}$, so $\mathrm{sgn}(\sum_{i=1}^{2n-1} \eta_i .\mathrm{sgn}(s_i)) = \mathrm{sgn}(\eta_{2n-1}.\mathrm{sgn}(s_{2n-1}))$ which means :

$$s_{2n-1}. \sum_{i=1}^{2n-1} \eta_i .\mathrm{sgn}(s_i) \geq 0$$

at same time, from the definition of the (12), we also know that: $\sum_{i=1}^{n} (c_{2i} \bar{d}_i - c_{2i} d_i) | \mathrm{sgn}(s_{2i-1}) | \geq 0$, Then, we can obtain

$$\lim_{t\to\infty} \int_0^t (k s^2_{2n-1} + s_{2n-1}. \sum_{i=1}^{2n-1} \eta_i .\mathrm{sgn}(s_i) + \sum_{i=1}^{n} (c_{2i} \bar{d}_i - c_{2i} d_i) | \mathrm{sgn}(s_{2i-1}) |) d\tau \leq v_{2n-1}(0) < \infty \quad (16)$$

According to Barbalat's lemma, we can know when $t \rightarrow \infty$,

$$k.s^2_{2n-1} + s_{2n-1}. \sum_{i=1}^{2n-1} \eta_i .\mathrm{sgn}(s_i) + \sum_{i=1}^{n} (c_{2i} \bar{d}_i - c_{2i} d_i) | \mathrm{sgn}(s_{2i-1}) |) \rightarrow 0 \quad (17)$$

Which means $\lim_{t\to\infty}(s_i = 0), \quad i = 1, 2, , ...n$ When i=n, the last layer sliding mode surface is asymptotically stable.

For the ith layer sliding surface, the Lyapunov function is $v_i = \frac{1}{2} s^2_i$ differentiating $v_i(t)$ with respect to time t obtains

$$\dot{v}_i = s_i.\dot{s}_i = s_i.[\sum_{j=1}^{m} c_{2j-1}.x_{2j} + \sum_{j=1}^{m} c_{2j} (f_j + b_j u + d_j)]$$

$$m = \begin{cases} \frac{i+1}{2} & i \quad odd \\ \frac{i}{2} & i \quad even \end{cases} \quad (18)$$

From (7), (11) and (12), we have $\sum_{j=1}^{i} \eta_j \,\mathrm{sgn}(s_j)$ and $\eta_i \,\mathrm{sgn}(s_i)$ are the same sign, so

$| \sum_{j=1}^{i} \eta_j \,\mathrm{sgn}(s_j) | < 2\eta_i \, | \mathrm{sgn}(s_i) | < \infty$, Defines $\sum_{j=1}^{i} \eta_j \,\mathrm{sgn}(s_j) = \eta'_i \,\mathrm{sgn}(s_i)$ where η is positive constant and we know that : $\eta'_i < 2\eta_i$,

$$\dot{v}_i = -\eta'_i \, | s_i | - \sum_{j=1}^{m} (c_{2j} \bar{d}_j - c_{2j} d_j) | \mathrm{sgn}(s_{2j-1}) | \leq 0$$

I ntegrating both sides

$$\int_0^t \dot{v}_i d\tau = \int_0^t -\eta_i^{'} |s_i| - \sum_{j=1}^{m} (c_{2j}\bar{d}_j - c_{2j} d_j) |\mathrm{sgn}(s_{2j-1})| d\tau \quad (19)$$

$$v_i(t) = v_i(0) - \int_0^t -\eta_i^{'} |s_i| d\tau - \sum_{j=1}^{m} (c_{2j}\bar{d}_j - c_{2j} d_j) |\mathrm{sgn}(s_{2j-1})| < \infty \quad (20)$$

$$\lim_{t\to\infty} \int_0^t -\eta_i^{'} |s_i| d\tau - \sum_{j=1}^{m} (c_{2j}\bar{d}_j - c_{2j} d_j) |\mathrm{sgn}(s_{2j-1})| \le v_i(0) < \infty \quad (21)$$

With using the Barbalat's lemma, there is $\lim_{t\to\infty}(s_i = 0), \quad i = 1,2,,...2n-2$ That is to say, all the sliding surfaces are asymptotically stable.

4. The GA Optimization

Genetic algorithms, introduced by Holland [4], are based on the idea of engendering new solutions from parent solutions, employing mechanisms inspired by genetics. In the following text the controller will be optimized by genetic algorithm. The optimization parameters are c_i and η_i. The number of variables is 8.

The fitness function is shown as formula. Evaluate the fitness value of each output.

$$j = \frac{1}{2}\sum_{k=1}^{n} (u_k - \hat{u}_k)^2 \quad (22)$$

u_k is expectation output and $\hat{u}_k$ is reality output, in this paper we have 8 Constant that designed with GA.

5. Ball and Beam System

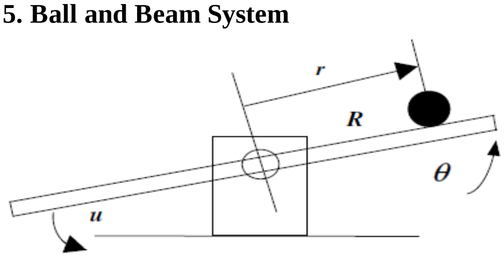


Fig 2. Ball and beam structure

This system is one off popular and important laboratory models for teaching control system engineering. These models cover many important modern and classical design methods. The ball moves freely along the length of the beam. Sensors are placed on one side of the beam to detect the position of the ball. An actuator must drive the beam to a desired angle, by applying a torque at the center. The control job is to automatically regulate the position of the ball by changing the position of the motor is very simple a steel ball rolling on the top of a long beam. It has a very important property open loop unstable, because the system output (the ball position) increases without limit for a fixed input (beam angle). This is a difficult control task because the ball does not stay in one place on the beam but moves with an acceleration that is proportional to the tilt of the beam [2].

According to the fig.2 and the position and velocity of the center of mass of the ball is:

$$r_c = (r.\cos(\theta), r.\sin(\theta))$$
$$v_c = (v.\cos(\theta) - r.\sin(\theta).\omega, v.\sin(\theta) + r.\cos(\theta).\omega) \quad (23)$$

Where $\dot{r} = v, \dot{\theta} = \omega$ the translational part of the kinetic energy is given by $k_{trans} = \frac{1}{2}mv_c^2 = \frac{1}{2}m(v^2 + r^2\omega^2)$ the rotational kinetic energy of the ball and beam system is: $k_{rot} = \frac{1}{2}j\omega^2 + \frac{1}{2}j_b(\frac{v}{R})^2$ where j is the inertia of the beam and jb the inertia of the ball, m is the mass of the ball, Hence, the total kinetic energy K = Ktrans + Krot equals:

$k = \frac{1}{2}(j + mr^2\omega^2) + \frac{1}{2}(1 + \frac{j_b}{mR^2})mv^2$, let $\lambda = (1 + \frac{j_b}{mR^2})$ is a constant. So we have $k = \frac{1}{2}(j + mr^2\omega^2) + \frac{1}{2}\lambda mv^2$, the potential energy of the ball and beam system is given by $v(x,\theta) = mgr\sin(\theta)$ the Lagrangian of the ball and beam system is as the following

$$L = \frac{1}{2}\begin{bmatrix}\dot{\theta} \\ \dot{r}\end{bmatrix}^T \begin{bmatrix} j + mr^2 & 0 \\ 0 & m\lambda \end{bmatrix} \begin{bmatrix}\dot{\theta} \\ \dot{r}\end{bmatrix} - mgr\sin(\theta) \quad (24)$$

The elements of the inertia matrix of the ball and beam system are

$$m_{11} = j + mr^2$$
$$m_{12} = 0$$
$$m_{22} = m\lambda$$

inertia matrix of the ball and beam system only depends on the position of the ball r. This means that the position of the ball is the shape variable of the ball and beam system which is unsaturated. Therefore, the ball and beam system is a Class (II) under actuated system.

$$(j+mr^2)\ddot{\theta}+2mr\dot{r}+mgr\cos(\theta)=\tau$$
$$m\lambda\ddot{r}-mr\dot{\theta}^2+mg\sin(\theta)=0 \quad (25)$$

$$\ddot{\theta}=\frac{\tau-2mr\dot{r}-mgr\cos(\theta)}{j+mr^2}$$
$$\ddot{r}=\frac{mr\dot{\theta}^2-mg\sin(\theta)}{m\lambda}=\frac{mR^2}{j_b+mR^2}(r\dot{\theta}^2-g\sin(\theta)) \quad (26)$$

$$\begin{Bmatrix} x_1=\theta \\ x_2=\dot{\theta} \\ x_3=r \\ x_4=\dot{r} \end{Bmatrix} \quad (27)$$

$$\dot{x}_1=x_2$$
$$\dot{x}_2=\frac{\tau-2mx_3x_4-mgx_3\cos x_1}{j+mx_3^2} \quad (28)$$
$$\dot{x}_3=x_4$$
$$\dot{x}_4=\frac{mR^2}{j_b+mR^2}(x_3x_2^2-g\sin x_1)$$

$$\dot{x}_1=x_2$$
$$\dot{x}_2=u+d \quad (29)$$
$$\dot{x}_3=x_4$$
$$\dot{x}_4=B(x_3x_2^2-g\sin x_1)$$

where $x_1=\theta$, the angle of the pole with respect to the vertical axis $x_2=\dot{\theta}$, the angle velocity of the pole with respect to the vertical axis $x_3=r$, the position of the cart $x_4=\dot{r}$, the velocity of the cart, B is the $\frac{mr^2}{j_b+mr^2}$, Jb is the moment of inertia of the ball; m is the mass of the ball; r is the radius of the ball; g is the acceleration of gravity.

6. SIMULATION RESULTS

In the simulation, the following specifications are used:

$B=0.7143, J_b=2\times10^{-6}, m=0.05kg,$

$r=0.1m, g=9.8m/s^2, |d|\le 0.08$

Initial values are:

$x_1=\theta=60^\circ, x_2=\dot{\theta}=0, x_3=r=10, x_4=\dot{r}=0$

With GA constant is found: c = [-1.636 -0.5672 0.3509 0.5165 0.0001 0.8015 1.054 0.71]

Fig.3. shows the entire sliding surfaces. By this control method, all sliding surfaces are asymptotically stable.

Figs. 4 and 5 show time responses of θ and r respectively. It is found that the ball and beam can be stabilized to the equilibrium point, and shown that θ and r converge to zero, respectively. Further, the performance and robustness of proposed control is better than reference [6] (Lon-Chen Hung, Hung-Yuan Chung2007).

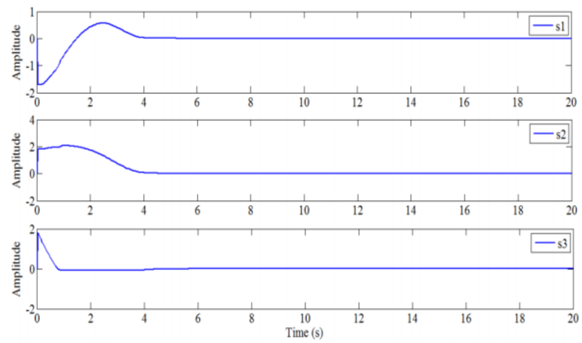
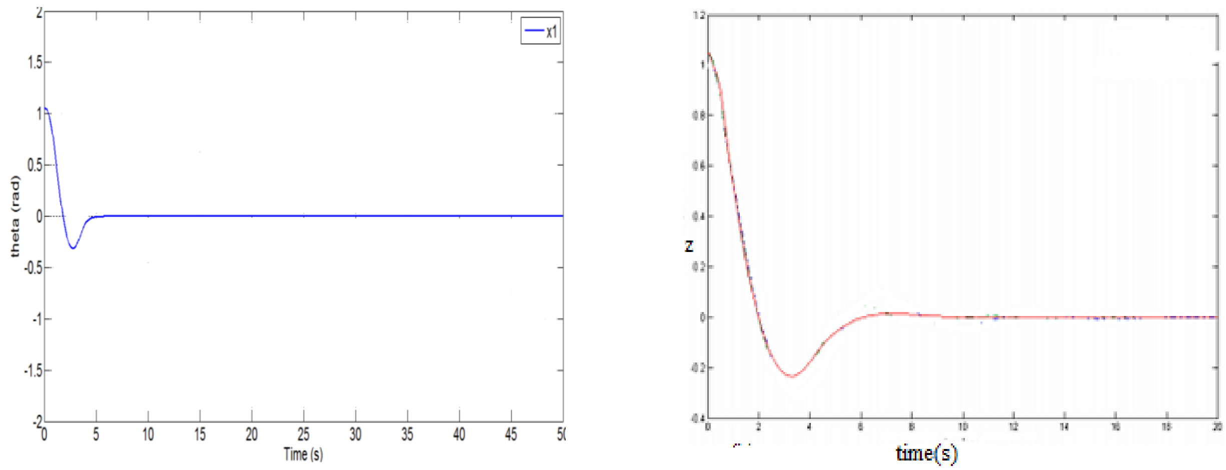


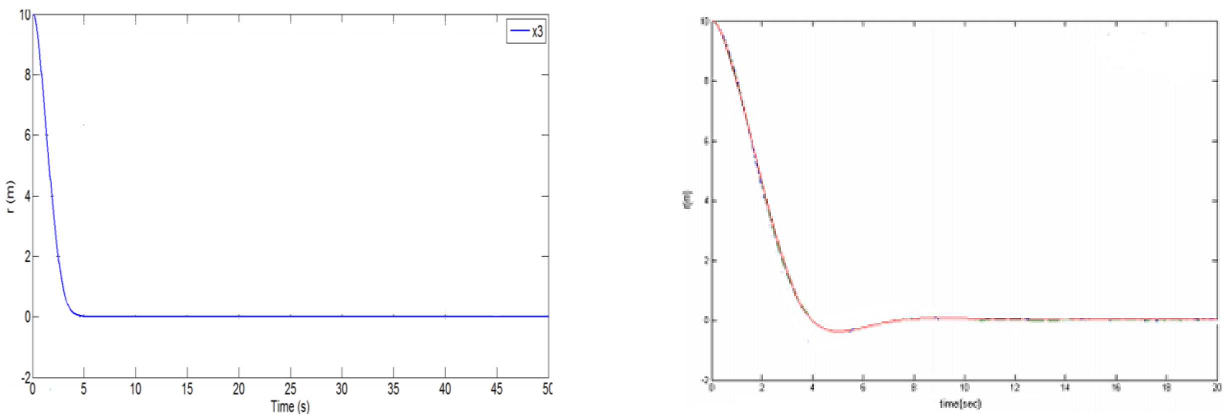
Fig 3. Sliding surfaces



(a) (b)

Fig 4. Angel evolution of the beam

a) Proposed control
b) Decoupled sliding-mode with fuzzy-neural network



(a) (b)

Fig 5. Position evolution of the ball

a) Proposed control
b) Decoupled sliding-mode with fuzzy-neural

7. Conclusions

The multiple layers sliding mode controller for a class of under actuated systems with mismatched uncertainties has been proposed in this paper. Simulation results were presented. The asymptotic stability of the entire sliding mode surfaces has been proved theoretically. For determining of the constants, genetic algorithm (GA) has been used. The proposed control method is applied to ball and beam system, the simulation results have shown that the proposed control better than Decoupled sliding mode with fuzzy neural network control (DSMFNNC) and the curves are smoother and the response time is shorter.

Corresponding author
Hamed Ebrahimi
Department of Electrical Engineering, Islamic Azad University (IAU), South Tehran Branch
Email: hamed_ebrahimi@aol.com

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